Matrices

The matrix: "Is an organization of some elements written in rows and columns between brackets in the form ( ) ".

Ex:

$$\begin{array}{cccc} \text{1st column} & \text{2nd} & \text{3rd} & \\ \left(\begin{array}{c} -5 \\ 1 \\ 0 \end{array}\right. & \begin{array}{c} 3 \\ 4 \\ \sqrt{3} \end{array} & \left.\begin{array}{c} 10 \\ -4 \\ 7 \end{array}\right) & \begin{array}{l} \to \text{1st row} \\ \to \text{2nd row} \\ \to \text{3rd row} \end{array} \end{array}$$

* The order of any matrix = no. of rows x no. of columns

How to express the elements in the matrix.

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$$

a_{32}: row ← → column

$\therefore$ a_{32} is the element in 3rd row and the 2nd column.

Ex:

Write the order of A, B and C then find:

a_{22} , a_{23} , a_{11} , b_{11} , b_{21} , b_{12} , c_{32} and c_{13}

$$A = \begin{pmatrix} 5 & 9 & -10 \\ -6 & \sqrt{5} & 9 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 8 \\ -3 & 17 \end{pmatrix}$$

and $$C = \begin{pmatrix} 2 & 1/4 & 8 \\ 0 & -1 & 14 \\ -2 & 9 & 1/5 \end{pmatrix}$$

Answer

- The order of A = 2 x 3

  $a_{22} = \sqrt{5}$, $a_{23} = 9$ $a_{11} = 5$

- The order of B =

  b_{11} = b_{21} = b12 =

- The order of C =

  C_{32}= C_{23} = C_{13}=

Some types of matrices:

1-The raw matrix

Ex: A = (3 2 -5) → 1 x 3 matrix

B = (34 -9) → 1 x 2 matrix

2-Column matrix

Ex: $X = \begin{pmatrix} 2 \\ -3 \\ \sqrt{2} \end{pmatrix}$ → 3 x 1 matrix

3- Squared matrix

Which no. Of rows = no. of column.

Ex: $Y = \begin{pmatrix} \sqrt{3} & 5 \\ -2 & 6 \end{pmatrix}$, $\begin{pmatrix} -1 & 3 & 1/5 \\ 2 & -6 & 8 \\ 5 & 8 & 4 \end{pmatrix}$

2 x 2 matrix 3 x 3 matrix

4-Zero matrix

All it is elements are zero

Ex: $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

2 x 3 2 x 1

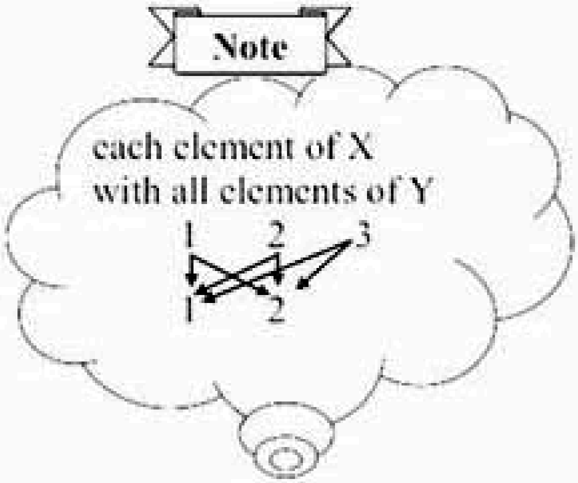
Ex1: Write the matrix (A_{xy}) of the dimensions 3×2 where

$a_{xy} = 2x - y$

Answer

row $X = 1, 2, 3$

column $Y = 1, 2$



$a_{11} = 2 \times 1 - 1 = 1$ $a_{12} = 2 \times 1 - 2 = 0$

$a_{21} = 2 \times 2 - 1 = 3$ $a_{22} = 2 \times 2 - 2 = 2$

$a_{31} = 2 \times 3 - 1 = 5$ $a_{32} = 2 \times 3 - 2 = 4$

$$\therefore A = \begin{pmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{pmatrix}$$

Ex2: Write the matrix (B_{xy}) of the order 3×3 where

$b_{xy} = 3x - 2y$

....

....

....

EX3: Write the matrix (B_{xy}) if the order 3×2 where

$bxy = x + 2y$

....

....

....

EX4: Write the matrix B = (b_{yz}) with the order 3 x 3 where

$$b_{yz} \begin{cases} Y + Z & \text{if } Y>Z \\ 0 & \text{if } Y = Z \\ Z - Y & \text{if } Y < Z \end{cases}$$

Answer

Y = 1 2 3

Z = 1 2 3

$b_{11} = 0$ $b_{12} = 2 - 1 = 1$ $b_{13} = 3 - 1 = 2$

$b_{21} = 1 + 2 = 3$ $b_{22} = 0$ $b_{23} = 3 - 2 = 1$

$b_{31} = 3 + 1 = 4$ $b_{32} = 3 + 2 = 5$ $b_{33} = 0$

$$B = \begin{pmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 4 & 5 & 0 \end{pmatrix}$$

Ex5: Write the matrix A = a_{xy} with the order 3 x 2 where

$$a_{xy} = \begin{cases} \text{Zero} & \text{if } X = Y \\ \\ X + Y & a < b \qquad \text{if } X \neq Y \end{cases}$$

Ex6: Write the matrix X = X_{ab} with the order 2 × 3 where

$$X_{ab} = \begin{cases} \text{Zero} & a = b \\ -1 & a > b \\ a + b & a < b \end{cases}$$

Transpose of matrix:

If A = (a_{xy}) then $A^T = a_{yx}$

Where A^T is the transpose of A

Note: $(A^T)^T = A$

Ex: Find the transpose of the following matrices and write its order:

$A \begin{pmatrix} 2 & 3 & 0 \\ -1 & 5 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 9 \\ -2 \\ 4 \end{pmatrix}$ and $C = \begin{pmatrix} -7 & 5 \\ 9 & 4 \end{pmatrix}$

$D \begin{pmatrix} 4 & 9 & -3 \\ 0 & 3 & 2 \\ {}^{1}/_{5} & -5 & {}^{-1}/_{3} \end{pmatrix}$ and $L = (4 - 1 \;\; 2)$

Answer

$A^d = \begin{pmatrix} 2 & -1 \\ 3 & 5 \\ 0 & 6 \end{pmatrix}$

3 X 2

$B^d = (9 \;\; -2 \;\; 4)$

1 x 3

$L^d = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$ 3 x 1

$C^d =$

$D^d =$

Verify that $(A^d)^d = A$

The equality of two matrices

If A and B are two matrices then A = B if and only if

1- A and B with the same order

2- The corresponding elements are equal.

Ex: $\begin{pmatrix} 1 & 0 & -2 \\ 2 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -2 \\ 2 & 3 & -1 \end{pmatrix}$

But: $\begin{pmatrix} 2 & 8 \\ 5 & -8 \end{pmatrix} \neq \begin{pmatrix} 2 & 5 \\ 8 & -5 \end{pmatrix}$ as ....

and $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \neq \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ as....................

Ex: Find the values of x, y and Z if

$$\begin{pmatrix} 7 & 0 & 2 \\ 4 & 7 & 5 \end{pmatrix} = \begin{pmatrix} -1 & 0 & X+5 \\ 4 & 2y-3 & 5 \end{pmatrix}$$

Answer

1- $\because$ The two matrices are equal

$\therefore Z = -1 \rightarrow (1)$

$2y - 3 = 7 \rightarrow (2)$

$X + 5 = 2 \rightarrow (3)$

$\because 2y - 3 = 7$

$2y = 7 + 3 = 10$

$2y = 10$

$y = \quad = 5$

$y = 5$

$\because X + 5 = 2$

$X = 2 - 5 = -3$

$X = -3$

Ex2: If $X = \begin{pmatrix} 3a+1 & 12-b & h^3 \\ c+2d & 18 & 6 \end{pmatrix}$

, $Y = \begin{pmatrix} 1 & 9 \\ 3 & 18 \\ -8 & d+2c \end{pmatrix}$

Find a , b , c , d and h if $X = Y^T$

Ex3: If $\begin{pmatrix} -8 & 15 \\ -1 & Z - y \end{pmatrix} = \begin{pmatrix} L^3 & X^2-1 \\ y & 8 \end{pmatrix}$

Find the value of X , y , z and **L**.

Ex4: Find the value of X , y and Z which make the two matrices

$\begin{pmatrix} X+2y & -1 \\ 5 & 2X + y \end{pmatrix}$ and $\begin{pmatrix} 3 & y - Z \\ 5 & 0 \end{pmatrix}$ equal

Ex5: Find the value of X, y which make $A = B^d$ where

$A = \begin{pmatrix} 1 & -3 & 6 \end{pmatrix}$

$B = \begin{pmatrix} 1 \\ -y \\ 3x \end{pmatrix}$

Symmetric and skew symmetric matrices:

If A is a square matrix , then

- A is called a symmetric matrix if and only if $A = A^T$
- A is called a skew symmetric matrix if and only if $A = -A^T$

$A = \begin{pmatrix} 2 & 1 & -3 \\ 1 & 4 & 0 \\ 3 & 0 & 5 \end{pmatrix}$ is symmetric matrix

$B = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & \frac{1}{2} \\ 2 & \frac{1}{2} & 0 \end{pmatrix}$ is skew symmetric

Sheet (1)

I- Complete:

1- If A is a matrix of order 2 x 2 and if $a_{11} = 5$, $a_{12} = 6$, $a_{12} = {}^1/_2$ and $a_{22} = \sqrt{5}$, then the matrix A = ……………

2- If X = $\begin{pmatrix} -1 & 3 & 2 \\ 2 & 4 & -3 \end{pmatrix}$, then the matrix X is of order………., X_{21} = …………………, X_{23} = ………………, X_{12}= …………..

3- If O is Zero matrix of order 2 x 2

$\therefore O^T$ = ………………………

4) X = $\begin{pmatrix} 2 & -2 & 1 \\ 3 & 5 & -3 \end{pmatrix}$, X = Y^T then y_{13} = ………. y_{31}=………

5) X = $\begin{pmatrix} 6 & 1 \\ 5 & -3 \end{pmatrix}$ = $\begin{pmatrix} 6 & 1 \\ X & -3 \end{pmatrix}$, then X = ………………………

X= $\begin{pmatrix} 4 & -2 & 3 \\ -3 & 0 & -1 \end{pmatrix}$, Y = $\begin{pmatrix} 6 & 0 & 1 \\ 4 & 2 & 3 \\ -7 & 8 & 6 \end{pmatrix}$

and Z = $\begin{pmatrix} {}^1/_2 \\ {}^1/_3 \\ -{}^1/_5 \end{pmatrix}$

1- Mention the order of each matrix.

2- Write each of the following elements X_{23}, y_{11}, Z_{31}, X_{31}, y_{33}, Z_{21}

3]-If X = $\begin{pmatrix} 2 & -3 & 5 \\ 0 & 2 & -4 \\ 7 & 2 & -8 \end{pmatrix}$ Check that $(X^T)^T = X$.

4]- If $\begin{pmatrix} 8 & 15 \\ 1 & Z+y \end{pmatrix} = \begin{pmatrix} a^3 & X^{2-1} \\ y & 4 \end{pmatrix}$

Then find the value of each X, y , Z.

5]- From the matrix $X = X_{ij}$ of order 3×3 where

$$X_{ij} = \begin{cases} i + j & \text{if } i > j \\ o & \text{if } i = j \\ j - i & \text{if } i < j \end{cases}$$

6]-From a matrix $y = (y_{ij})$ of order 3×2

$Y_{ij} = i - j + 2$

Then find the matrix X where $X = Y^{t}$

mention its order and find X_{ij} if $i=3j$.

7] show which of the following is symmetric and which is skew symmetric :

a) $\begin{pmatrix} 0 & 2 & 4 \\ 2 & 0 & 5 \\ 4 & 5 & 0 \end{pmatrix}$ b) $\begin{pmatrix} 0 & 3 & 2 \\ 3 & 0 & 5 \\ 2 & 5 & 0 \end{pmatrix}$

8] If $A = \begin{pmatrix} 5 & 2x & 8 \\ 4 & 3 & 6 \\ x+2y & 6 & 4 \end{pmatrix}$ is a symmetric matrix , then Find the value of : x , y

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9] If $B = \begin{pmatrix} 0 & 3x & 7 \\ x+3 & 0 & -2z \\ 3y-x & 6 & 0 \end{pmatrix}$ is skew symmetric matrix Find the value of x , y and z

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Operation on matrices

I-Addition:

To add two matrices A, B they must have the same order.

Ex1: If A = $\begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$, B = $\begin{pmatrix} 6 & -7 \\ 4 & 3 \end{pmatrix}$

A + B = $\begin{pmatrix} 2+6 & 3+(-7) \\ -1+4 & -2+3 \end{pmatrix} = \begin{pmatrix} 8 & -4 \\ 3 & 1 \end{pmatrix}$

Ex2: If A = $\begin{pmatrix} 2 & -2 \\ 4 & 6 \end{pmatrix}$

$\therefore$ 3A = $\begin{pmatrix} 3\times 2 & 3\times -2 \\ 3\times 4 & 3\times 6 \end{pmatrix} = \begin{pmatrix} 6 & -6 \\ 12 & 18 \end{pmatrix}$

Properties of addition of matrices:

Let A, B, C matrices of order m x n, o_{mxn} be a zero matrix of the sam order.

1-Closure property:

A+B is a matrix of order mxn

2-Commutative property:

A+B = B+A

Ex: $\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 5 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 5 & 4 \end{pmatrix}$

3-Associative property:

(A+B) + C = A + (B+C)

Ex: If A $\begin{pmatrix} 3 & 2 \\ 1 & -4 \end{pmatrix}$, B = $\begin{pmatrix} -1 & 1 \\ 4 & -3 \end{pmatrix}$, C = $\begin{pmatrix} 5 & 1 \\ -3 & 2 \end{pmatrix}$

Then (A + B) + C =

$$\left[\begin{pmatrix}3 & 2\\1 & -4\end{pmatrix} + \begin{pmatrix}-1 & 1\\4 & -3\end{pmatrix}\right] + \begin{pmatrix}5 & 1\\-3 & 2\end{pmatrix}$$

$$= \begin{pmatrix}2 & 3\\5 & -7\end{pmatrix} + \begin{pmatrix}5 & 1\\-3 & 2\end{pmatrix} = \begin{pmatrix}7 & 4\\2 & -5\end{pmatrix}$$

$$A + (B + C) = \begin{pmatrix}3 & 2\\1 & -4\end{pmatrix} + \left[\begin{pmatrix}-1 & 1\\4 & -3\end{pmatrix} + \begin{pmatrix}5 & 1\\-3 & 2\end{pmatrix}\right]$$

$$\begin{pmatrix}3 & 2\\1 & -4\end{pmatrix} + \begin{pmatrix}4 & 2\\1 & -1\end{pmatrix} = \begin{pmatrix}7 & 4\\2 & -5\end{pmatrix}$$

4-Identity of addition:

$$A + O = O + A = A$$

Ex: $A + O = \begin{pmatrix}2 & 5\\3 & 6\end{pmatrix} + \begin{pmatrix}0 & 0\\0 & 0\end{pmatrix} = \begin{pmatrix}2 & 5\\3 & 6\end{pmatrix} = A$

5-Additive inverse:

$$A + (-A) = (-A) + A = 0$$

Where – A is the additive inverse of A

$$A + (-A) = \begin{pmatrix}3 & -4 & 2\\1 & 0 & -5\end{pmatrix} + \begin{pmatrix}-3 & 4 & -2\\-1 & 0 & 5\end{pmatrix} = \begin{pmatrix}0 & 0 & 0\\0 & 0 & 0\end{pmatrix} = 0$$

Ex1: If $A\begin{pmatrix}3 & 1\\2 & -1\end{pmatrix}$, $B = \begin{pmatrix}-1 & 3\\4 & 6\end{pmatrix}$, $C = \begin{pmatrix}0 & 5\\-2 & 4\end{pmatrix}$

Find: (1) A + B (2) B – C (3) A + 2B – C

Answer

(1) $A + B = \begin{pmatrix}3 & 1\\2 & -1\end{pmatrix} + \begin{pmatrix}-1 & 3\\4 & 6\end{pmatrix} = \begin{pmatrix}3+(-1) & 1+3\\2+4 & -1+6\end{pmatrix} = \begin{pmatrix}2 & 4\\6 & 5\end{pmatrix}$

(2) $B - C = B + (-C) = \begin{pmatrix}-1 & 3\\4 & 6\end{pmatrix} + \begin{pmatrix}0 & -5\\2 & -4\end{pmatrix} = \begin{pmatrix}-1 & -2\\6 & 2\end{pmatrix}$

(3) $A + 2B - C = \begin{pmatrix} 3 & 1 \\ 2 & -1 \end{pmatrix} + 2\begin{pmatrix} -1 & 3 \\ 4 & 6 \end{pmatrix} - \begin{pmatrix} 0 & 5 \\ -2 & 4 \end{pmatrix}$

$= \begin{pmatrix} 3 & 1 \\ 2 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 6 \\ 8 & 12 \end{pmatrix} + \begin{pmatrix} 0 & -5 \\ 2 & -4 \end{pmatrix}$

$= \begin{pmatrix} 3+(-2)+0 & 1+6+(-5) \\ 2+8+2 & -1+12+(-4) \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 \\ 12 & 7 \end{pmatrix}$

$= A + 2B + (-C)$

<u>Ex2</u>: If $X = \begin{pmatrix} 5 & -1 \\ 1 & 3 \\ 0 & 2 \end{pmatrix}$, $Y \begin{pmatrix} 2 & -3 \\ 4 & 6 \\ 5 & 1 \end{pmatrix}$, $Z - \begin{pmatrix} 1 & -1 \\ 2 & -3 \\ -4 & 3 \end{pmatrix}$

Find (1) $2X - 3Y + Z$ (2) $3X - 2y + 3Z$

(3) $X - 3Y - Z$

<u>Ex3</u>: If $A = \begin{pmatrix} 3 & 4 & 1 \\ 5 & 6 & -1 \\ 4 & 1 & -2 \end{pmatrix}$, $B \begin{pmatrix} 2 & 4 & 1 \\ 3 & -4 & 1 \\ 1 & 5 & 0 \end{pmatrix}$

Find the matrix X such that $A - X = 2B$

Answer

$\because A - X = 2B$

$\therefore -X = 2B - A$

$\therefore X = -2B + A = A + (-2)\,B$ Find X by your self.

Ex4: If $\begin{pmatrix} 5 & 1 \\ 2 & -3 \end{pmatrix} - \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 2 & -1 \end{pmatrix}$

Find a , b , c , d

Answer

$$\begin{pmatrix} 5-a & 1-b \\ 2-c & -3-d \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 2 & -1 \end{pmatrix}$$

$\therefore 5 - a = 4$

$-a = 4 - 5 = -1$ $\therefore \boxed{a = -1}$

$1 - b = 3$

$-b = 3 - 1 = 2$

$b = 2$

$-3 - d = -1$

$-d = -1 + 3 = 2$

$\boxed{d = -2}$ Find C.

Ex5: If $A = \begin{pmatrix} 4 & 6 & -1 \\ 2 & 3 & 1 \\ 7 & -2 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -1 & 4 \\ 6 & 5 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

Find the matrix X if $X = 3A - 4B$

Ex6: If $B = \begin{pmatrix} 3 & 5 & 2 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

Find the matrix X which satisfies the equation

$2B + 3C = 4X - 3D$.

Ex7: If $A = \begin{pmatrix} 3 & 5 & 1 \\ 2 & -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 2 & 1 \\ 0 & 3 & -1 \end{pmatrix}$

Find the matrix X such that: $2B + X^T = A$

$2B + X^T = A$

$X^T = A - 2B$

$$\therefore X^T = \begin{pmatrix} 3 & 5 & 1 \\ 2 & -1 & 0 \end{pmatrix} - 2\begin{pmatrix} -1 & 2 & 1 \\ 0 & 3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 5 & 1 \\ 2 & -1 & 0 \end{pmatrix} + \begin{pmatrix} 2 & -4 & -2 \\ 0 & -6 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 & -1 \\ 2 & -7 & 2 \end{pmatrix}$$

$$\therefore (X^T)^T = \begin{pmatrix} 5 & 2 \\ 1 & -7 \\ -1 & -2 \end{pmatrix} = X$$

Ex8: If $X^T + \begin{pmatrix} 2 & 1 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Find the matrix X

Ex9: If $A = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 0 & 2 \\ 4 & -3 & -2 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 1 & 4 \\ 2 & 1 & -3 \\ 0 & 3 & -1 \end{pmatrix}$

Find the matrix X if $A + 3X = 2B$

Sheet (2)

I-Complete:

1) I A + $\begin{pmatrix} -3 & -2 \\ 5 & 4 \end{pmatrix}$ = 0 , then A = ………………………………

2) If O is the Zero matrix of order 2 x 2 , then 4O = ……………and it is of order………………………

3) If each of the matrices A and B is of order 3 x 1, then the resultant matrix of A – 2B is of order……………………

4) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}^{T}$ = ………………… which is of order……………

5) If A = $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, then 3 A = ………………, - 2A = ……………

6) If A = $\begin{pmatrix} 15 & 10 \\ 5 & 20 \end{pmatrix}$, then A = 5 $\begin{pmatrix} \cdots & \cdots \\ \cdots & \cdots \end{pmatrix}$

II- If A = $\begin{pmatrix} -3 & 1 \\ 2 & 5 \end{pmatrix}$ and B = $\begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$

Check that: 1) $(A + B)^{T} = A^{T} + B^{T}$

2) $A - B \neq B - A$

III- If A = $\begin{pmatrix} 1 & 0 & 3 \\ 3 & 2 & -2 \\ 5 & 6 & 0 \end{pmatrix}$ and $(A + B)^{T}$ = $\begin{pmatrix} 6 & 3 & 7 \\ 3 & -1 & 5 \\ 4 & 5 & 4 \end{pmatrix}$

Find the matrix B.

IV- If $\begin{pmatrix} 3 & 6 \\ 5 & -7 \end{pmatrix} + \begin{pmatrix} 1 & -4 \\ -2 & 6 \end{pmatrix} = \begin{pmatrix} X & 4 \\ 7 & Y \end{pmatrix}$

Find the value of X and Y.

V- Find X, Y, Z, and L that satisfy that:

$$X\begin{pmatrix} 1 & 3 \\ 5 & Y \end{pmatrix} + Z\begin{pmatrix} 2 & L \\ 0 & 4 \end{pmatrix} + 5\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = O_{2\times 2}$$

VI- If $A = \begin{pmatrix} 5 & -3 & 6 \\ 2 & 5 & 0 \\ 4 & -2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 1 & 3 \\ -4 & 21 & -5 \\ 3 & 12 & 6 \end{pmatrix}$

Find the matrix X such that:

1) $3A + X = 2B$ 2) $2A + B - X = 0$

3) $2A - X = B^T$

VII- If $X + 2X^T = \begin{pmatrix} 9 & 14 \\ 13 & 6 \end{pmatrix}$, find the matrix X.

VIII- If $A = \begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix}$ and $B^T = \begin{pmatrix} -1 & 0 \\ 3 & 4 \end{pmatrix}$, find the matrix X such that: $4X - 3B + 2A^T = A + (5B - X)^T$.

Multiplying Matrices

- If A is a matrix of order mxn, B is a matrix of order r x L, then their product C = A x B will be defined if and only if n = r
- To multiply two matrices A no. of columns= no. or rows B

2 x 3 ——— 3 x 1

2x1 is the order of the product matrix

Ex: If A= $\begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix}$, B = $\begin{pmatrix} -2 & 1 \\ 5 & 6 \end{pmatrix}$

$$AB = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix} \cdot \begin{array}{cc} 1 & 2 \\ \multicolumn{2}{c}{\begin{pmatrix} -2 & 1 \\ 5 & 6 \end{pmatrix}} \end{array}$$

$$\begin{pmatrix} (4 \times -2) + (-3 \times 5) & (4 \times 1) + (-3 \times 6) \\ (2 \times -2) + (-1 \times 5) & (2 \times 1) + (-1 \times 6) \end{pmatrix} = \begin{pmatrix} -23 & -13 \\ -9 & -4 \end{pmatrix}$$

Ex: If A $\begin{pmatrix} 3 & -2 \\ 0 & 2 \\ -1 & 4 \end{pmatrix}$, B = $\begin{pmatrix} 3 & -1 \\ 5 & 7 \end{pmatrix}$, C = $\begin{pmatrix} 4 & 0 & 3 \\ 5 & 2 & -1 \end{pmatrix}$

and D = $\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

Check that: 1) $(AB)^T = B^T A^T$

2) (AB)C = A(BC)

3) (B + D) C = BC + DC.

4) $D^2 - 4D + 4I = (D - 2I)^2$

Answer

$$1)\ (AB) = \begin{pmatrix} 3 & -2 \\ 0 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 9-10 & -3-14 \\ 0+10 & 0+14 \\ -3+20 & 1+28 \end{pmatrix} = \begin{pmatrix} -1 & -17 \\ 10 & 14 \\ 17 & 29 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} -1 & 10 & -17 \\ -17 & 14 & 29 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 3 & 5 \\ -1 & 7 \end{pmatrix} \begin{pmatrix} 3 & 0 & -1 \\ -2 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 9-10 & 0+10 & -3+20 \\ -3-4 & 0+14 & 1+28 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 10 & -17 \\ -17 & 14 & 29 \end{pmatrix}$$

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2) $$(AB)\,C = \begin{pmatrix} -1 & -17 \\ 10 & 14 \\ 17 & 29 \end{pmatrix} \begin{pmatrix} 4 & 0 & 3 \\ 5 & 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -4-85 & 0-34 & -3+17 \\ 40+70 & 0+28 & 30-14 \\ 68+145 & 0+58 & 51-29 \end{pmatrix}$$

$$= \begin{pmatrix} -89 & -34 & 14 \\ 130 & 28 & 16 \\ 213 & 58 & 22 \end{pmatrix}$$

$$A\,(BC) = \begin{pmatrix} 3 & -2 \\ 0 & 2 \\ -1 & 4 \end{pmatrix} \left[\begin{pmatrix} 3 & -1 \\ 5 & 7 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 & 3 \\ 5 & 2 & -1 \end{pmatrix} \right]$$

$$= \begin{pmatrix} 3 & -2 \\ 0 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 12-5 & 0-2 & 9+1 \\ 20+35 & 0+14 & 15-7 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -2 \\ 0 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 7 & -2 & 10 \\ 55 & 14 & 8 \end{pmatrix} = \begin{pmatrix} -89 & -34 & 14 \\ 110 & 28 & 16 \\ 213 & 58 & 22 \end{pmatrix}$$

Sheet (3)

Review on unit (1)

I-Complete:

1-If $\begin{pmatrix} -7 \\ x \\ 2 \end{pmatrix} - 2 \begin{pmatrix} Y \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ -2 \end{pmatrix}$, then X = ..., Y = ...

2- If I is the identity matrix of order 2 x 2 then $(I^2)t$ = ...

3- If $\begin{pmatrix} 4 & -X \\ -1 & y \end{pmatrix} = \begin{pmatrix} 4 & Z \\ 0 & 3 \end{pmatrix}^T$ Then X = ..., Y = ..., Z=........

4-If A is a matrix of order 1 x 3, B is a matrix of order.............. then AB is a matrix of order 1 x 2.

5- $\begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix} = -2 \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix}$

6- If A = $\begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}$, B = $\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$, then A + 3B =.........................

7-If I is the identity matrix of order 3 x 3, then 3I = ...

8- If A = $\begin{pmatrix} 1 & 4 & 2 \\ 0 & -6 & 8 \end{pmatrix}$, then A^t = ..., A^t is of order = ...

9-If the matrix A is of order m x n, B is a matrix of order L x K, then the required condition that makes AB defined is............in this case the matrix AB is of order................

10- $\begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ 3 & 3 \end{pmatrix}$ = ...

11- If $\begin{pmatrix} 2 & -1 \\ 4 & y^2 \end{pmatrix} + \begin{pmatrix} Z & 0 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & X \\ 6 & 3 \end{pmatrix}$, then X = ... ,

y=............., Z=.................

12- $\begin{pmatrix} 3 & -1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \\ 6 \end{pmatrix}$ = ...

13- If $\begin{pmatrix} 1 & 3 \\ 2 & -4 \\ 6 & -2 \end{pmatrix} \times X = 2 \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}$ then X = ...

14- $\begin{pmatrix} -3 & -1 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 2 & 0 \end{pmatrix}$ = ...

15- If $\begin{pmatrix} 3 & -1 & X^2 \\ 4 & 5 & 2 \end{pmatrix} + \begin{pmatrix} 2 & Z & 2 \\ Y & 3 & 0 \end{pmatrix} = \begin{pmatrix} 5 & -1 & 6 \\ 6 & 8 & 2 \end{pmatrix}$

Then X = ..., y = ..., Z = ...

16- If A = $\begin{pmatrix} -1 & 5 & 4 \\ 3 & 2 & 0 \end{pmatrix}$, then $(A^T)^T$ = ...

17- $\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix} + 2 \begin{pmatrix} -1 & -1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ = ...

18- If X + $\begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}$ = 0

II- a) If C = $\begin{pmatrix} -1 & 1 \\ -3 & 2 \\ 2 & -1 \end{pmatrix}$, Q^T = $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, A=(-1 1 0)and B = $\begin{pmatrix} 4 & 2 & -1 \\ 5 & 3 & -1 \\ 3 & -1 & 4 \end{pmatrix}$ Prove that AB = QC^T.

b) IA = $\begin{pmatrix} 2 & 2 \\ 1 & 0 \end{pmatrix}$, B = $\begin{pmatrix} 4 & 0 \\ 3 & -1 \end{pmatrix}$, C = $\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$

$Q = \begin{pmatrix} -2 & 3 \\ 3 & -4 \end{pmatrix}$ Prove that $C\,(A + B^T)\,Q = I$

c) Solve the matrix equation: $2\left[X^T + \begin{pmatrix} 3 & 2 \\ -1 & 5 \end{pmatrix}\right] = X^T - 4I$

d) If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 5 \\ 7 & -8 \end{pmatrix}$

Prove that: $(AB - BA)^2 + 1052\,I = 0$

e) If $A = \begin{pmatrix} 3 & 4 & 2 \\ -3 & 5 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 2 \\ 3 & 4 \\ -2 & -8 \end{pmatrix}$ and $C = \begin{pmatrix} 12 & 5 \\ 6 & -4 \end{pmatrix}$

Prove that: $AB - C^T = 2I$

F) If: $A = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -2 \end{pmatrix}$, Find the matrix X such that $X^T = (AB)^2$

g) If $A = \begin{pmatrix} X & 1 \\ 3 & 0 \end{pmatrix}$ and $(A^T)^2 - 2A^T - 3I = 0$

h) If A and B are two matrices and if $BA = \begin{pmatrix} 1 & 0 & -5 \\ 2 & 3 & -2 \\ 5 & -1 & -4 \end{pmatrix}$

Find $A^T B^T$.

i) If $L = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$, $M = \begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}$ and $Q^T = \begin{pmatrix} 4 & 3 \\ 0 & -1 \end{pmatrix}$

find the matrix: $L\,(M^T - Q)$

k) $A = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix}$, n is a realnumber

Prove that: $(A + nB^2)^3 = 16nI$ where I is the identity matrix of order 3 x 3.

L) If A = $\begin{pmatrix} 1 & 2 & 0 \\ 5 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, B^T = $\begin{pmatrix} 1 & -10 & 0 \\ -4 & -5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Find the value of X where (X – 2) I = 2A + B

m) If A $\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$ prove that $A^3 - 3A^2 + 3A - I = (A- I)^3$

n) If $(A^T - B)^T$ = $\begin{pmatrix} 4 & -2 \\ 3 & 1 \end{pmatrix}$, B = $\begin{pmatrix} -2 & -1 \\ 0 & 6 \end{pmatrix}$ find the matrix A.

o) If A = $\begin{pmatrix} 1 & 2 & 0 \\ 5 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, B^T = $\begin{pmatrix} 1 & -10 & 0 \\ -4 & -5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Find the value of X where (X – 2) I = 2A + B.

p) A^T = $\begin{pmatrix} 4 & 3 \\ 7 & 6 \end{pmatrix}$, B = $\begin{pmatrix} 6 & -7 \\ -3 & 4 \end{pmatrix}$ Prove that $(AB)^3 = 27I$.

r) If A = $\begin{pmatrix} -3 & 0 \\ 4 & 1 \end{pmatrix}$, B = $\begin{pmatrix} 2 & 5 \\ -1 & 4 \end{pmatrix}$, C = $\begin{pmatrix} 0 & 4 \\ -6 & 2 \end{pmatrix}$

and if 2 $[X^T - AB] = 3C^T$ find the matrix.

S) If X = $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 4 & 4 \\ 3 & -5 & 1 \end{pmatrix}$, Y^T = $\begin{pmatrix} 1 & 0 & -3 \\ -2 & -2 & 5 \\ 3 & -4 & 1 \end{pmatrix}$

Prove that: XY = YX

T) A = $\begin{pmatrix} 3 & 0 & 2 \\ -1 & 4 & 0 \\ 1 & 2 & -2 \end{pmatrix}$, B = $\begin{pmatrix} -1 & 0 & -2 \\ 1 & -2 & 0 \\ -1 & -2 & 4 \end{pmatrix}$

Prove that: $A^T + B^T = 2I$

V) If A = $\begin{pmatrix} 2 & -3 \\ 1 & -1 \end{pmatrix}$, Prove that $A^3 + I = 0$

Determinants

- second order

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Find the value of the following determinant :

a) $\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}$ b) $\begin{vmatrix} 4 & 7 \\ 2 & 6 \end{vmatrix}$ c) $\begin{vmatrix} 5 & 4 \\ 3 & 2 \end{vmatrix}$ d) $\begin{vmatrix} \sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{vmatrix}$

- Third order

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & j \end{vmatrix} = a\begin{vmatrix} e & f \\ h & j \end{vmatrix} - b\begin{vmatrix} d & f \\ g & j \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= -a\begin{vmatrix} e & f \\ h & j \end{vmatrix} + b\begin{vmatrix} d & f \\ g & j \end{vmatrix} - c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

- Find the value of the following determinant :

a) $\begin{vmatrix} 4 & 1 & 3 \\ 0 & 5 & 2 \\ 0 & 3 & 1 \end{vmatrix}$ b) $\begin{vmatrix} 1 & 2 & 5 \\ 0 & 3 & 2 \\ 0 & 0 & 6 \end{vmatrix}$

- solve the equation : $\begin{vmatrix} x & 0 & 1 \\ x & 1-x & x \\ x & 1 & 1+x \end{vmatrix} = 0$

- Find the area of a triangle whose vertices are X(1,2) ,Y(3,-4) and Z(-2,3)

- Prove using determinants that the point (-2,4) , (3,0) And (8,-4) are collinear.

- Solve the system of the following equation using Cramer's Rule : $4x+3y=-4$, $3x-y=-2$

Multiplicative inverse of a matrix

1] Show the matrix which have multiplicative inverse :

a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ c) $\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix}$

d) $\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$ e) $\begin{pmatrix} 1 & 0 \\ 3 & 4 \end{pmatrix}$ f) $\begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$

2] what is the real values of a which make each of the following matrices has A multiplicative inverse :

a) $\begin{pmatrix} a & 1 \\ 6 & 3 \end{pmatrix}$ b) $\begin{pmatrix} a & 9 \\ 4 & a \end{pmatrix}$

3] if : $X = \begin{pmatrix} 1 & x \\ 0 & x \end{pmatrix}$ prove that : $X^{-1} = X$

4] solve each of the following system using the matrices :

a) $3x+2y=5$, $2x+y=3$

b) $2x-7y=3$, $x-3y=2$

5] use the matrices to find the two numbers in which their sum equal 10 and The difference between them equal 4.

Solving inequatities of first

Degree in one variable and in two variables

Show: graphically the solution set of the inequality 3X+10>1, then write in the form of an interval where $X \in R$

Solution:

$3X + 10 > 1$

$3X > 1 - 10$

$3X > -9 \div 3$

$X > -3$

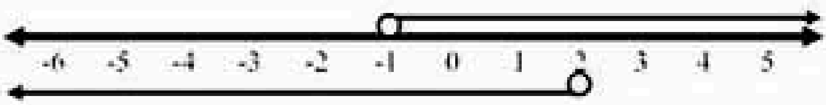
S.S = $]-3, \infty[$

2) $2X - 2 \leq 3X - 1 < X + 5$

$2X - 2 \leq 3X - 1$	$3X - 1 < X + 5$
$2X - 3X \leq -1 + 2$	$3X - X < 5 + 1$
$-X \leq 1 \div (-1)$	$2X < 6 \div 2$
$X \geq -1$	$X < 3$
S.S = $[-1, \infty[$	S.S = $]-\infty, 3[$

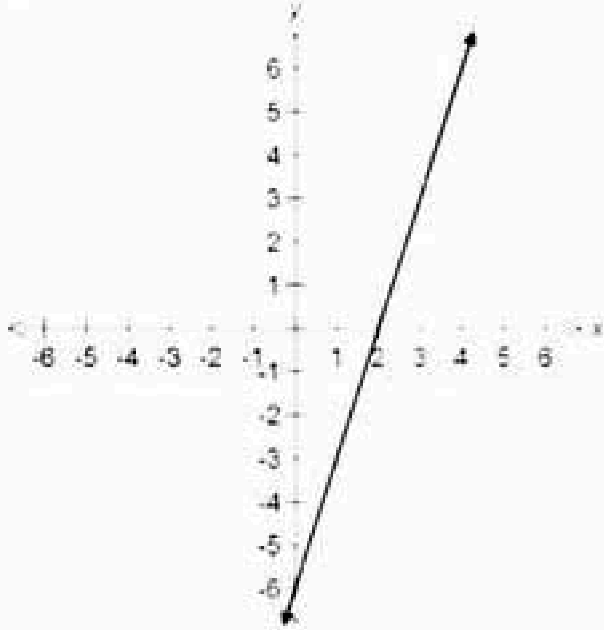
S.S. of the original inequality

$= [-1, \infty[\cap]-\infty, 3[= [-1, 3[$



3- Solve the inequality graphically:

a) $3X - Y \leq 6$



Step (1): $3X - Y = 6$

Put $X = 0$

Step (2):

Put $Y = 0$

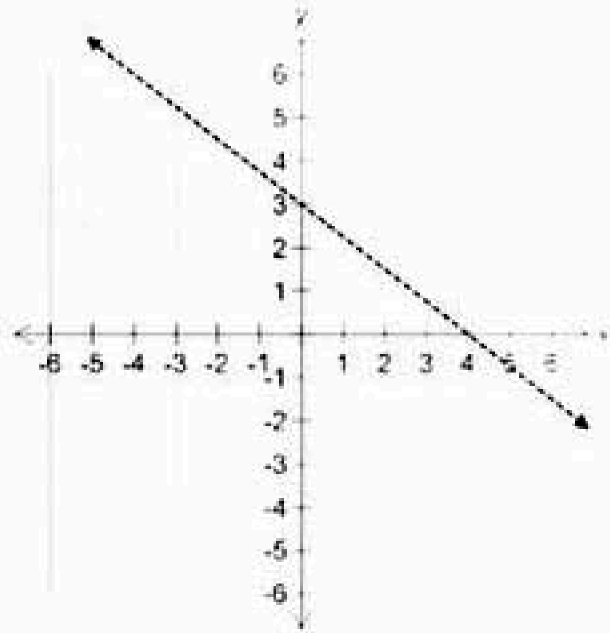
Step (3): Graph.

Step (4): Substituted by (0,0)

$0 < 6$

X	0	2
Y	-6	0

b) $3X + 4Y < 12$



Step (1): $3X + 4Y = 12$

Step (2):

Step (3):

Test (0 , 0)

$0 < 12$

X	0	4
Y	3	0

Note:

1-The equation: $Y = 0$ is represented by the X – axis.

2-The equation: $X = 0$ is represented by the Y – axis.

3-The equation: $Y = a$ is represented by a straight line parallel to the x- axis and passes through the point $(0 , a)$.

4-The equation $X = a$ is represented by straight line parallel to y-axis and passes through the point $(a, 0)$.

5-The straight line whose equation is in the form: $\frac{X}{a} + \frac{Y}{b} = 1$ passes through two points (a, o) and (o , b).

Ex: Represent graphically the S.S of the two inequalities $X - 2y \leq 1$. $X + 2y \leq 3$

Answer

Let $X - 2y = 1$

Test $(0 , 0)$

$0 < 1$

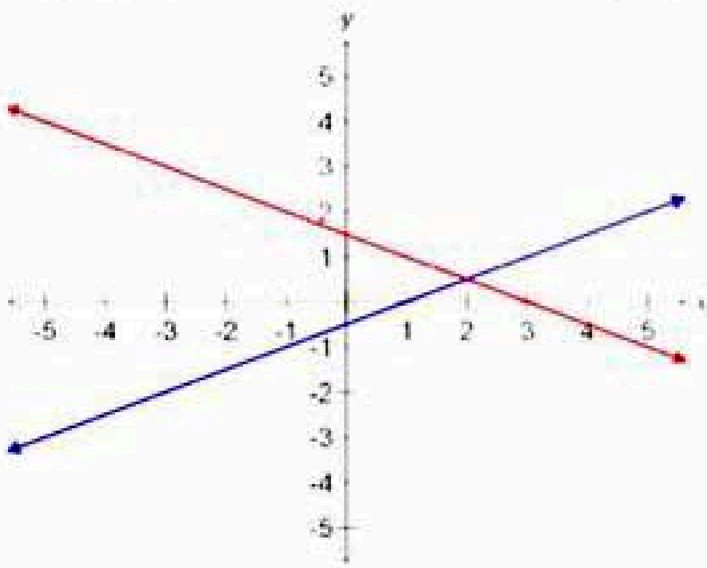
X	0	1
Y	$-^1/_2$	0

Let $X + 2y = 3$

Test $(0 , 0)$

$0 < 3$

X	0	3
Y	$-^3/_2$	0



S.S is colloured region $S = S_1 \cap S_2$

Find graphically the S.S of each of the following:

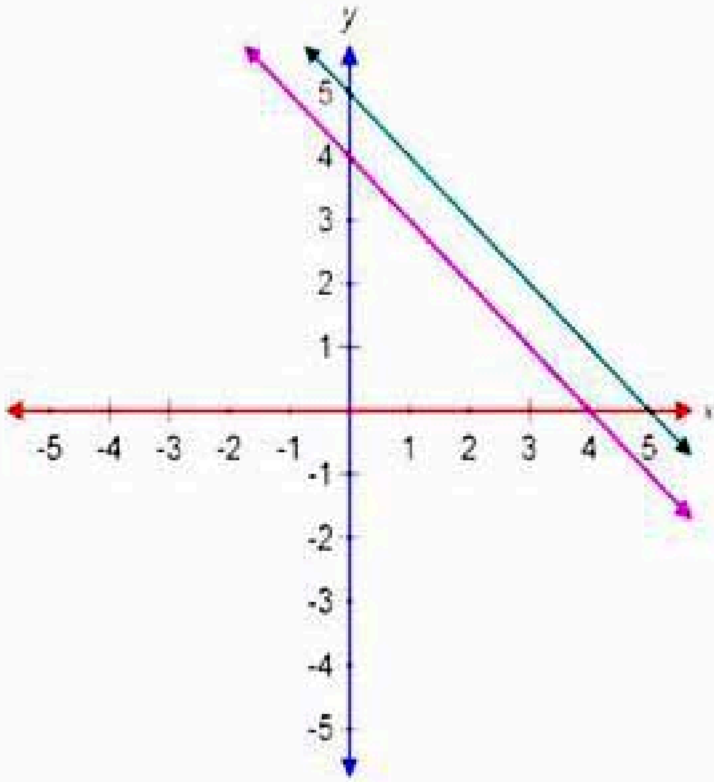
$X \geq 0$, $y \geq 0$, $X + y \leq 5$, $X + Y \leq 4$

Answer

L1: X = 0

L2: y = 0

L3 : X + y = 5



X	0	5
Y	5	0

Test + (0 , 0) O <5

L4: X + y = 4

X	0	4
Y	4	0

Test (0 , 0)

0 < 4

S.S is coloured region.

Linear programming

Find graphically the S.S of the set of inequalities:

1- $X \geq 0$, $y \geq 0$, $y - X \leq 3$ and $2y + 5X \leq 20$

Then find from the S.S the value of (X,y) that makes R maximum value where $R = 5X + 3y$

Answer

L1: $X = 0$,

L2: $y = 0$

L3: $y - X = 3$

Test + (0 , 0)

$O < 3$

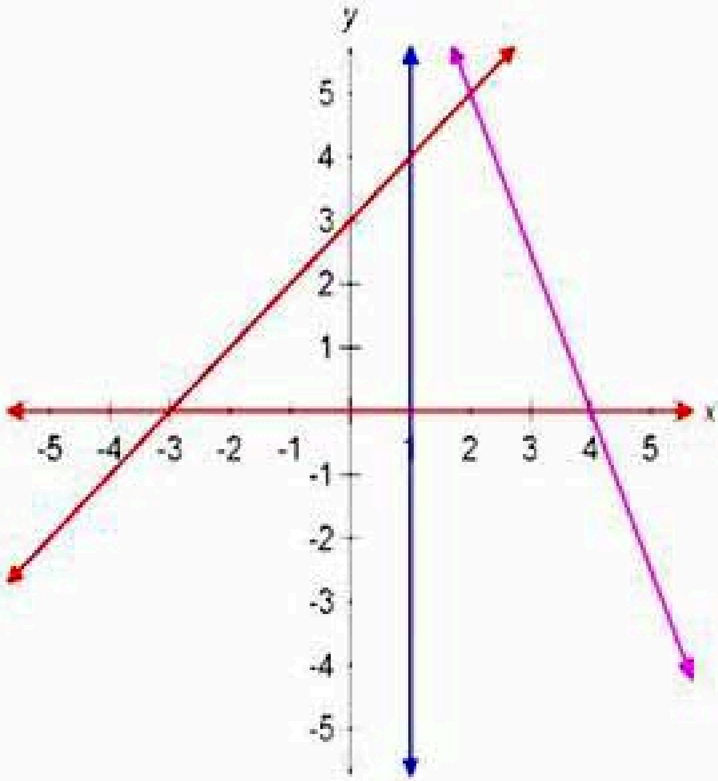
X	0	-3
Y	3	0

L4: $2y + 5X = 20$

Test + (0 , 0)

$O < 20$

X	0	4
Y	10	0



Points of intersection are (0 , 0), (4, 0), (0, 3), (2, 5) sub in R at (0,0)

5X + 3y = 5 X 0 + 3 X 0 = 0 at (4 , 0)

Max value at (2, 5). 20 + 0 = 20 0 + 9 = 9

At (2 , 5) 10 + 15 = 25 Mx.

Ex2: Find the greatest and smallest value of the expression L = X+3y-5 on the region that satisfies the following condition:

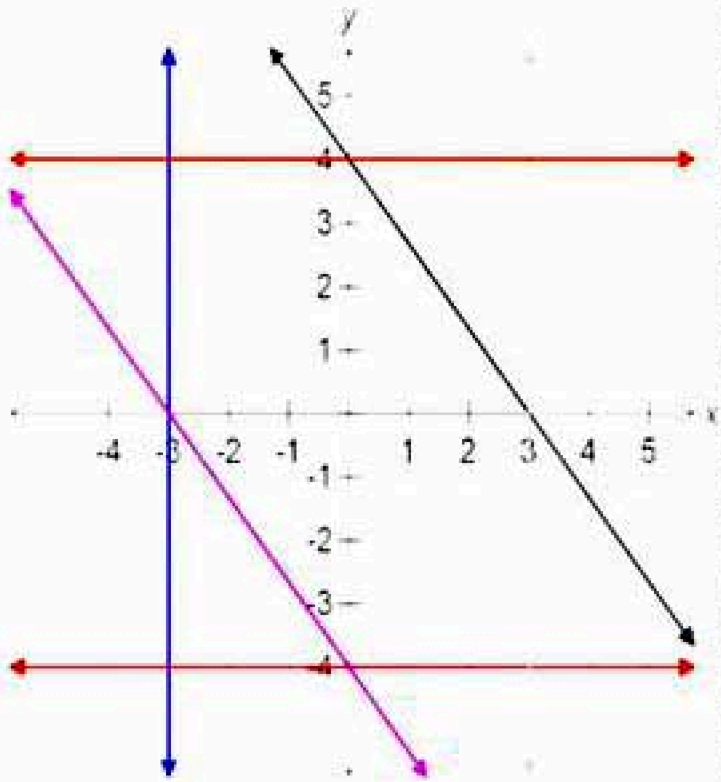
$-3 \leq X \leq 3$; $-4 \leq y \leq 4$, $4X + 3y \leq 12$ and $4X + 3y \geq -12$

L3: 4X + 3y = 12

X	0	3
Y	4	0

L4: 4X + 3y = -12

X	0	-3
Y	-4	0



L = X + 3y – 5

at (3 , 0) : L = 3+0–5 = -2

at (3 , -4) L=3–12 –5 = -14

at (0 , -4) L =0–12 – 5=-17

at (-3 , 0) L = -3 – 5= -8

at (-3 , 4) L =-3+12–5= 4

at (0 , 4) L = 0 + 12 – 5 = 7

Smallest value is – 17

Greatest value is 7.

Ex: A factory produces two kinds of accessories (A) and (B) producing one piece of the first kind (A) needs two machines. The first machine works for one hour second machine works for 2hours and half and producing one piece from the 2^{nd} kind (B) needs the first machine to work for 4hours and the second machine for 2hours if the first machine works for 8hours. At most daily and the second machine works 12hours at most daily. The profit of the factory is L.E. 24 and L.E 40 from each piece of the kind (A) and (B) respectively. Find the maximum profit that the factory can make in one day.

Answer

1-Summarize the data in the problem in the following table:

	(A)	(B)	The most no. of hours
First machine needs no. of hours	1	4	8
Second machine needs no. of hours.	$2^1/_2$	2	12
The profit	24	40	

2- Translate the data in the form of inequalities.

1) $X \geq 0$, $y \geq 0$ 2) $X + 4y \leq 8$

3) $^5/_2X + 2y \leq 12$ X 2

$5X + 4y \leq 24$

The objective function. $R = 24\ X + 40y$.

3- Represent the inequalities graphically.

L1: $X + 4y = 8$

X	0	8
Y	2	0

Test (0 , 0)

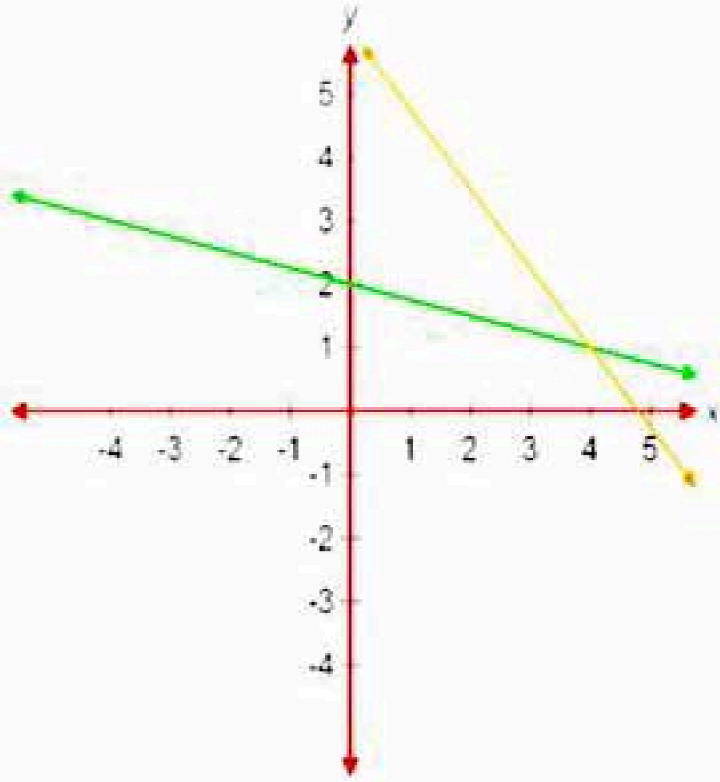
$0 < 8$

L2: $5X + 4y = 24$

X	0	4.8
Y	6	0

Test (0 , 0)

$0 < 24$



A = (4.8,5) , B = (4,1), C = (0,2), D (0, 0)

$\because$ The objective function $R = 24X + 40y$

$R_A = 24 \times 4.8 + 0 = 115.2$

$R_B = 24 \times 4 + 40 \times 1 = 136$

$R_C = 0 + 40 \times 2 = 80$

R_D = Zero.

The maximum profit = 136 when the factory produces 4 units from kind A and one unit from kind B.

Review sheet (4)

On linear programming

I- Choose:

1-In the S.S of the inequalities:

$X \geq 0$, $y \geq 0$, $2X + y \geq 8$, $X + 2y \geq 10$ the point (….,….) is the point that makes the objective function $R = 2X+y$ be maximum.

a) (4,4) **b) (10,0)** **c) (0,8)** **d) (2,4)**

2-The point………………lies in the region of the s.s of the inequality $3X + 4y \geq 8$.

a) (0,1) **b) (-2,2)** **c) (1,1)** **d) (2,1)**

3-In the S.S of the inequalities $X \geq 0$, $y \geq 0$, $X + y \leq 4$, $2X + y \leq 5$ we find that (…,…). Makes the objective function $R=6X+4y$. give the maximum value.

a) (2.5, 0) **b) (1,3)** **c) (0,4)** **d) (2,1)**

4-In the S.S of the inequalities $X \geq 0$, $y \geq 0$, $X+ y \geq 3$, $X+ 2y \geq 5$ we find (…,…) makes the value of the objective function $R = 4X + y$ minimum.

a) (1,2) **b) (2,3)** **c) (0,3)** **d) (5,0)**

5- The point …………..belongs to S.S of the inequality $X \geq 2$.

a) (0 , 0) **b) (2 , 4)** **c) (-2 , 4)** **d) (-2 , 0)**

6-The point (3, -2) belongs to the S.S of the inequality: 2X+y ……………...5.

a) $\geq$ **b) $\leq$** **c) >** **d) =**

II- Complete:

1-The point (0,0) belongs to the S.S of the inequality X + 3y …6.

2-(2,4) belongs to the S.S of the inequalities X ………-1, y……1

3-(0,0) belongs to the S.S of the inequality: y- X ...3.

4-(4,0) belongs to the S.S of the inequality: y........3X.

5-(4,-4) belongs to the S.S of the inequality: y + 2 X ......Zero.

III- Find graphically the S.S. of the two inequalities:

1) $y \leq X$, $\boxed{X < 3}$ < 1 simultaneously.

2) $y \leq X$, $X - 2y < 1$ simultaneously.

3) Find graphically the S.S of $y \geq 2$, $y \geq -3X$

4) $y \leq X$, $X - 2y > 1$

5) $X \geq 0$, $0 \leq y$, $y \leq X$, $X + y \leq 10$

6) $y \leq 1$, $y \geq -3X$

IV- a) A carpentry work shop produces two kinds A and B. Each table is made by two persons, a carpenter and a painter. The carpenter needs one hour to make the table of the kind A while he needs two hours to make the table of kind B the painter needs two hours to paint the table of kind (A) and one hour to paint the table of kind (B). the carpenter works for 8 hours daily at most while the painter works for 10 hours daily at most. If the work shop sells all it daily production with profit L.E 30 to the table of kind A and L.E 40 to the table of kind B.

How many tables of each kind the work shop should produce daily to achieve maximum profit.

b) A baby home decided to offer a light meal to babies the meal consists of 2 kinds of pies such that the meal given to each child contains 4 units at least of vitamin A and 9 units at least of vitamin B. If we assume that the pie of the first kind gives at average one unit of vitamin A and 3 units of vitamin B and the price of the pie of the 1^{st} kind is one pound and the price of the pie of the 2^{nd} kind 1.5 pound. find how many pies of each kind needed to make the meal cheaper and guarantees. The Lowest limit of vitamins.

Remember that (Algebra):

- **Solving Quadratic Equations:** $ax^2 + bx + c = 0$

  By the general formula: $X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- **Vertex of Quadratic Equations :** $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

- **Complex number:** $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

- **The order of the matrix = m x n**

  where m is the number of rows and n is the number of columns.

- **Some special matrices:**

  * **Row matrix:** one row and any number of columns
  * **Column matrix:** one column and any number of rows
  * **Square matrix:** number of rows = the number of columns
  * **The Zero "Null" matrix** $O_{m \times n}$: all elements are zeros
  * **Diagonal matrix:** It is a square matrix in which all elements are zeros except the elements of its diagonal at least one of them is not equal to zero.
  * **Unit matrix:** it is a diagonal matrix in which each element on the main diagonal has the numeral 1, while all other elements = 0

- **Equality matrices:** Two matrices A and B are equal if and only if they have the same order and the corresponding elements are equal: $a_{ij} = b_{ij}$, $\forall$ i and j

- **Matrix Transpose:**

  A of order m x n → A^t of order n x m

  Notice that: $(A^t)^t = A$ & $(AB)^t = B^t A^t$

- **Symmetric Matrices:** $A = A^t$, (A square matrix)
- **Skew Symmetric Matrices:** $A = -A^t$ (A square matrix)
- $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a\,d - b\,c$
- $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(e\,i - f\,h) - b\,(d\,i - f\,g) + c\,(d\,h - e\,g)$

Notice that: The sign of the minor determinant same as the sign of

$(-1)^{i+j}$ as follow: $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

- **Determinant of triangular Matrix:**

$$\begin{vmatrix} a & 0 & 0 \\ d & e & 0 \\ g & h & i \end{vmatrix} \text{ or } \begin{vmatrix} a & b & c \\ 0 & e & f \\ 0 & 0 & i \end{vmatrix} = a\,e\,i$$

Equals the product of the elements of its principal diagonal

- **Area of Δ ABC: X(a , b), Y(c , d), Z(e , f) = |A| where:**

$$A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$$

Notice that: if area = zero, then A B and C are collinear

• Cramer's rule

- **Solving a system of Linear equations in two variables:**

$$a\,x + b\,y = m\ ,\ c\,x + d\,y = n,$$

Let $\Delta \neq 0$, then the solution of the system is:

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix},\ \Delta x = \begin{vmatrix} m & b \\ n & d \end{vmatrix},\ \Delta y = \begin{vmatrix} a & m \\ c & n \end{vmatrix} \therefore X = \frac{\Delta x}{\Delta}\ ,\ Y = \frac{\Delta y}{\Delta}$$

$$\therefore S.S = \{(x\ ,\ y)\}$$

Solving a system of Linear equations in three variables:

$$a_1 x + b_1 y + c_1 z = m,\ a_2 x + b_2 y + c_2 z = n,\ a_3 x + b_3 y + c_3 z = k$$

Let $\Delta \neq 0$, then the solution of the system is:

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},\ \Delta x = \begin{vmatrix} m & b_1 & c_1 \\ n & b_2 & c_2 \\ k & b_3 & c_3 \end{vmatrix},$$

$$\Delta y = \begin{vmatrix} a_1 & m & c_1 \\ a_2 & n & c_2 \\ a_3 & k & c_3 \end{vmatrix},\ \Delta z = \begin{vmatrix} a_1 & b_1 & m \\ a_2 & b_2 & n \\ a_3 & b_3 & k \end{vmatrix},$$

$$\therefore X = \frac{\Delta x}{\Delta}\ ,\ Y = \frac{\Delta y}{\Delta}\ ,\ Z = \frac{\Delta z}{\Delta}$$

$$\therefore S.S = \{(x\ ,\ y\ ,\ z)\}$$

- **The multiplicative inverse of the matrix A:**

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ let } \Delta \neq 0\ ,\ \Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \text{ then:}$$

$$A^{-1} = \frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Notice that: $A A^{-1} = A^{-1} A = I$

- **Solving a system of Linear equations in two variables using Inverse Matrix:**

$a_1x + b_1y = k_1$, $a_2x + b_2y = k_2$ then: $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$

$$A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}, X = \begin{pmatrix} x \\ y \end{pmatrix}, C = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \text{ Then:}$$

$$AX = C \quad \therefore X = A^{-1} C, \ |A| \neq 0$$

Remember that (Trigonometry):

- The area of the circle is $= \pi r^2$
- The circumference of the circle is $= 2\pi r$
- The central angle: $\theta^{rad} = \frac{\ell}{r}$ or $\ell = \theta^{rad} \times r$

$r \rightarrow$ is the Radius of this circle

$\ell \rightarrow$ the length of the arc

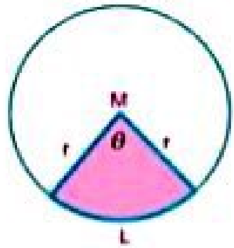
- Relation between degree measure and radian measure:

$$\frac{x^o}{180^o} = \frac{\theta^{rad}}{\pi}$$

$\theta^{rad} \rightarrow$ is the radian measure

$X^\circ \rightarrow$ is the degree measure

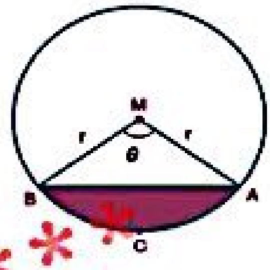
- Area of the circular sector $= \dfrac{\theta^{o}}{360}\pi r^2$



$$= \frac{1}{2} r^2 \theta^{rad}$$

$$= \frac{1}{2} \ell r$$

- Area of the circular segment $= \dfrac{1}{2} r^2 (\theta^{rad} - \sin\theta)$



- Area of the triangle $= \dfrac{1}{2}$ AB x BC Sin B

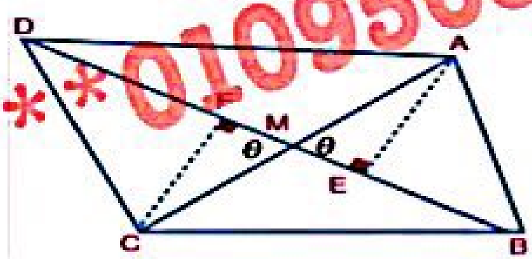
$$= \frac{1}{2} \text{ AB x AC Sin A}$$

$$= \frac{1}{2} \text{ AC x CB Sin C}$$

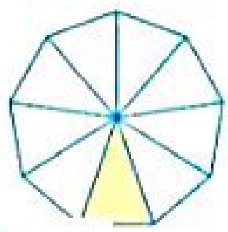
$= \dfrac{1}{2}$ the lengths of two sides x sine the included angle between them.

- The Area of a Convex Quadrilateral $= \dfrac{1}{2} D_1 \times D_2 \sin\theta$

$= \dfrac{1}{2}$ lengths of its diagonals x sine the included angle between them

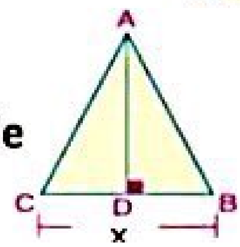


- The area of a regular polygon $= \dfrac{1}{4} n X^2 \cot\dfrac{\pi}{n}$

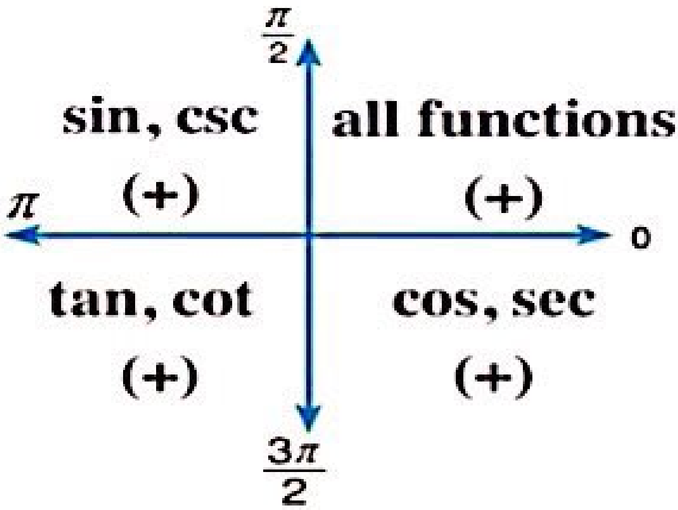


Note: AD $= \dfrac{1}{2} x \cot\dfrac{\pi}{n}$

<u>where</u>: n is number of sides , x length of its side



- **Summary of signs of all trigonometric ratios:**



- **Basic Trigonometric Identities**

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta \quad \& \quad \cos^2\theta = 1 - \sin^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta \;\rightarrow\rightarrow\rightarrow\; \sec^2\theta - \tan^2\theta = 1$$

$$1 + \cot^2\theta = \csc^2\theta \;\rightarrow\rightarrow\rightarrow\; \csc^2\theta - \cot^2\theta = 1$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

- **General solution of trigonometric equations:**

When: $\sin(\alpha) = \cos(\beta)$, then $\alpha \pm \beta = \frac{\pi}{2} + 2\pi n,\ n \in Z$

When: $\csc(\alpha) = \sec(\beta)$, then $\alpha \pm C = \frac{\pi}{2} + 2\pi n,\ \ n \in Z$

$$\alpha \neq n\pi,\ \beta \neq (2n+1)\frac{\pi}{2}$$

When: $\tan(\alpha) = \cot(\beta)$, then $\alpha + \beta = \frac{\pi}{2} + \pi n,\ n \in Z$,

$$\alpha \neq (2n+1)\frac{\pi}{2},\ \beta \neq n\pi$$

- **Trigonometric functions of angles of measures**

$$\theta, (180^\circ \pm \theta)$$

$\text{Sin}\,(180^\circ + \theta) = -\text{Sin}\,\theta$, $\text{Sin}\,(180^\circ - \theta) = \text{Sin}\,\theta$

$\text{Cos}\,(180^\circ + \theta) = -\text{Cos}\,\theta$, $\text{Cos}\,(180^\circ - \theta) = -\text{Cos}\,\theta$

$\text{Tan}\,(180^\circ + \theta) = \text{Tan}\,\theta$, $\text{Tan}\,(180^\circ - \theta) = -\text{Tan}\,\theta$

- **Trigonometric functions of angles of measures**

$$\theta, (360^\circ - \theta)$$

$\text{Sin}\,(360^\circ - \theta) = -\text{Sin}\,\theta$, $\text{Csc}\,(360^\circ - \theta) = -\text{Csc}\,\theta$

$\text{Cos}\,(360^\circ - \theta) = \text{Cos}\,\theta$, $\text{Sec}\,(360^\circ - \theta) = \sec\theta$

$\text{Tan}\,(360^\circ - \theta) = -\text{Tan}\,\theta$, $\text{Cot}\,(360^\circ - \theta) = -\text{Cot}\,\theta$

- **Trigonometric functions of angles of measures**

$$\theta, (90^\circ \pm \theta)$$

$\text{Sin}\,(90^\circ + \theta) = \text{Cos}\,\theta$, $\text{Sin}\,(90^\circ - \theta) = \text{Cos}\,\theta$

$\text{Cos}\,(90^\circ + \theta) = -\text{Sin}\,\theta$, $\text{Cos}\,(90^\circ - \theta) = \text{Sin}\,\theta$

$\text{Tan}\,(90^\circ + \theta) = -\text{Cot}\,\theta$, $\text{Tan}\,(90^\circ - \theta) = \text{Cot}\,\theta$

- **Trigonometric functions of angles of measures**

$$\theta, (270^\circ \pm \theta)$$

$\text{Sin}\,(270^\circ + \theta) = -\text{Cos}\,\theta$, $\text{Sin}\,(270^\circ - \theta) = -\text{Cos}\,\theta$

$\text{Cos}\,(270^\circ + \theta) = \text{Sin}\,\theta$, $\text{Cos}\,(270^\circ - \theta) = -\text{Sin}\,\theta$

$\text{Tan}\,(270^\circ + \theta) = -\text{Cot}\,\theta$, $\text{Tan}\,(270^\circ - \theta) = \text{Cot}\,\theta$

- In the function f where $f(\theta)$ = Sin θ then:

  The domain is $]-\infty, \infty[$

  The range is $[-1, 1]$

  The cosine function is periodic with period 2π

The maximum value = 1 and takes place at the points

$$\theta = \frac{\pi}{2} \pm 2n\pi, \quad n \in Z$$

The minimum value = -1 and takes place at the points

$$\theta = \frac{3\pi}{2} \pm 2n\pi, \quad n \in Z$$

- In the function f where f(θ) = Cos θ then:

  The domain is: $]-\infty, \infty[$,

  The range is: $[-1, 1]$

  The cosine function is periodic with period 2π

The maximum value = 1 and takes place at the points

$$\theta = \pm 2n\pi, \quad n \in Z$$

The minimum value = -1 and takes place at the points

$$\theta = \pi \pm 2n\pi, \quad n \in Z$$

Final math revision second Term secondary1_2019

First: Complete the following questions

1) The area of Δ ABC where: A(9 , 4) , B(0 , 16) , C(0 , 0) equals……….

2) The area of Δ ABC in which: AB = 8 cm , BC = 6 cm and m($\angle$B) = 30° equals……….

3) If: A = $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$, B = $\begin{pmatrix} 2 & 5 \end{pmatrix}$ then $(AB)^t$ =……….

4) $\begin{vmatrix} 8 & 5 \\ 7 & 3 \end{vmatrix}$ =…………….

5) If each of the matrices A and B is of order 3 x 1, then the resultant matrix of A – 2B is of order……

6) If A is a matrix of order 1 x 3, B is a matrix of order…………… then AB is a matrix of order 1 x 2.

7) If the matrix A is of order m x n, B is a matrix of order L x K, then the required condition that makes AB defined is………….

8) If: A = $\begin{pmatrix} 3 & 2 & 1 \\ -1 & 0 & 0 \end{pmatrix}$, B = $\begin{pmatrix} 2 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$ then: A – B =………………..

9) $\begin{vmatrix} 3 & 0 & 0 \\ 5 & -2 & 0 \\ 4 & 1 & 2 \end{vmatrix}$ =.................

10) If: A = $\begin{pmatrix} 1 & 4 & 2 \\ 0 & -6 & 8 \end{pmatrix}$ then A^t =..........

11) If the matrix $\begin{pmatrix} a & 2i \\ 2i & -1 \end{pmatrix}$ has a multiplicative inverse

then: a =.............

12) If: A = $\begin{pmatrix} -1 & 2 & 5 \\ 3 & 0 & 4 \end{pmatrix}$ then A^t of order..........

13) If: $\begin{pmatrix} x^2 - 3 & 2 \\ 5 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 5 & 1 \end{pmatrix}$ Then: x =............

14) If: $\begin{pmatrix} 15 & 5 \\ 20 & 10 \end{pmatrix} = 5\begin{pmatrix} 3a & 1 \\ 4 & 2 \end{pmatrix}$ Then: a =...........

15) If: A = $\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}$, B = $\begin{pmatrix} 0 & 4 \\ 2 & 6 \end{pmatrix}$ Then: BA =...........

16) If: A = $\begin{pmatrix} 1 & 2 \\ 3 & -2 \end{pmatrix}$ Then: A^2 =................

17) $A^{-1} = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$ Then: A =..............

18) A = $\begin{pmatrix} -1 & 0 \\ 8 & -2 \end{pmatrix}$ Then: A^{-1} =..............

19) If: $\begin{pmatrix} x + 3 \\ y - x \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ Then: x =......... , y =..........

20) If: $\begin{pmatrix} 4 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -3 & x \end{pmatrix}$ = I Then x =..........

21) The area of circular sector whose circle radius length $= 6$ cm and its central angle 30° is.........

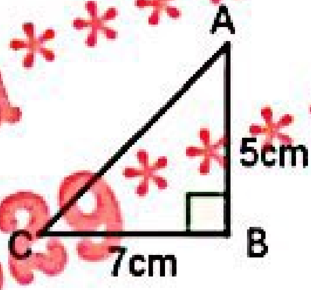
22) The area of the regular pentagon whose side length $= 10$ cm equals............ To nearest tenth

23) The area of the quadrilateral whose diagonals lengths 12 cm , 8 cm and the measure of the concluded angel between them $= 30^\circ$ is.........

24) A circular sector whose perimeter $4r$ cm where r is the radius length of its circle then its central angle measure =.........

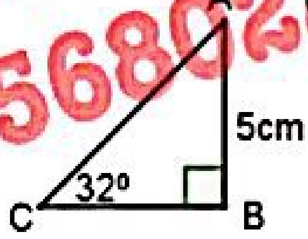
25) In the opposite figure:

$m(\angle C)$ =..............



26) In the opposite figure:

BC =....... To nearest cm



27) The area of the triangle ABC in which $m(\angle A) = 48^\circ$ $AB = 9$ cm, $AC = 12$ cm, to the nearest hundredth.......

28) If: $\sec\theta + \tan\theta = 4$, then: $\sec\theta - \tan\theta$ =........

29) $\dfrac{\sin(\frac{\pi}{2}-\theta)}{\cos(\frac{\pi}{2}-\theta)}$ =....................

30) The two points (4 , 3) and (3 , 2) $\in$ the S.S of the inequality $x + y$ ……… 5

31) If: $\tan^2\theta = 5$, then $\sec^2\theta =$…………

32) In Δ ABC, if AB = AC = 10 cm , $m(\angle B) = 30^\circ$, then its area =……………….. cm^2

33) The simplest form of:

$$(\cos\theta - \sin\theta)^2 + 2\sin\theta\cos\theta = \ldots\ldots\ldots\ldots$$

34) The simplest form of: $\sin^2\theta + \cos^2\theta + \cot^2\theta =$…….

35) The area of equilateral triangle of side length 10 cm equals………………

36) The general solution of: $\cos\theta = \sin\theta$ is…………

37) If: $\cot\theta = 2$, then : $\csc^2\theta =$……….

38) $\sec^2 7\theta - \tan^2 7\theta =$…………….

39) If: $\sin\theta\cos\theta = \frac{1}{10}$, then: $(\sin\theta - \cos\theta)^2 =$………

40) $(\sin^2\theta + \cos^2\theta)^9 =$…………

41) $7\sin^2\theta + 7\cos^2\theta =$…………

42) If: $A - A^t = \square$ then A is called…………

43) A circular sector whose perimeter 25 cm and the length of its arc = 7 cm then its area =………

44) The area of the circular segment in which the length of the radius of its circle is 7 cm and its height 3 cm is……………..

45) If the matrix $\begin{pmatrix} a & 6 \\ 2 & a-1 \end{pmatrix}$ has a multiplicative inverse then: a =…..

46) If the matrix $\begin{pmatrix} x-1 & -2 \\ 1 & x-1 \end{pmatrix}$ has a multiplicative inverse then: x =…..

47) The area of the minor circular segment in which the length of its chord is 12 cm, and its height is 2 cm. to the nearest cm^2 is…………………………

48) The S.S of the equation: 4 $\sin^2 \theta = 3$ is…………
Where $\theta \in [0, 2\pi[$

49) The S.S of the equation: 3 $\sec^2 \theta = 4$ is…………
Where $\theta \in [0, 2\pi[$

50) If: $5^{\sin\theta} = \frac{1}{25}$, $\theta \in [0, 2\pi]$, then the S.S =………

51) If: $3^{\cos\theta} = 1$, $\theta \in [0, 2\pi]$, then the S.S =………

52) The perimeter of the circular sector =………………..

Second: Choose the correct answer.

1) If: $\begin{vmatrix} 2x & 2 \\ 4 & 3 \end{vmatrix} = 10$, then: x =...... (2 , 3 , 4 , 5)

2) If: A is a matrix of order 3 x 3 , then the number of elements of the matrix A is.... (3 , 6 , 9 , 12)

3) If: A is a matrix of order 2 x 3 , B^t is a matrix of order 1 x 3 then the order of the matrix AB is.......

(3 x 3 , 3 x 1 , 2 x 1 , 1 x 2)

4) If the matrix $\begin{pmatrix} a & 8 \\ 2 & a \end{pmatrix}$ has no multiplicative inverse then: a =..... (4 , ± 4 , $\in R - \{4\}$, $\in R - \{-4 , 4\}$)

5) If: A = $(2 \quad 3)$, B = $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ then the possible operation from the following is.......

(A + B , $A^t + B^t$, AB , $A B^t$)

6) If: $A + A^t = \square$ then A is.....

(symmetric , skew symmetric , row matrix , column matrix)

7) The S.S of the two equations: 2x – 3y = 1 , 3x + 2y =8 is..... ((3 , 2) , (1 , 2) , (2 , 1) , (2 , 3))

8) The point ....belongs to S.S of the following inequalities X > 2 , y > 1 and x + y ≥ 3 is......

((3 , 2) , (1 , 2) , (2 , 1) , (1 , 3))

9) The point ....belongs to S.S of the following inequalities $X \geq 0$, $y \geq 0$, $2x + y < 4$ and $x + 3y < 6$ is...... ((3 , 0) , (1 , −3) , (2 , 1) , (2 , 3))

10) The point ....belongs to S.S of the following inequality: $y < 2x + 3$ is...........

((−1 , −1) , (−1 , 1) , (−3 , −3) , (0 , 3))

11) The point ....$\notin$ the S.S of: $2x - y \leq 7$ in R x R is..... ((0 , 0) , (2 , 0) , (3 , −2) , (5 , 4))

12) The point at which the function $P = 40x + 20y$ has a maximum value is..... ((3 , 1) , (1 , 2) , (3 , 2) , (1 , 3))

13) The point at which the function $P = 35x + 10y$ has a minimum value is..........

((0 , 0) , (0 , 10) , (0 , 40) , (20 , 10))

14) If the perimeter of circular sector = 10 cm and the length of its arc = 2 cm then its area =..... cm^2

(20 , 10 , 8 , 4)

15) If the area of circular sector = 110 cm^2 and its central angle = 2.2^{rad} then the radius length of their circle =..... cm (20 , 10 , 2 , 5)

16) The area of equilateral triangle of 6 cm length equals........ cm^2 ($6\sqrt{3}$, $9\sqrt{3}$, $12\sqrt{3}$, $18\sqrt{3}$)

17) If the area of circular sector = 4 cm^2 and its arc length = 2 cm then their perimeter =..... cm

(20 , 10 , 8 , 6)

18) The S.S of the equation: Cos θ + Sin θ = 0 is.....° ,

$180° < \theta < 360°$ ({210} , {225} , {240} , {315}

19) If: $0° < \theta < 360°$, Sin θ + 1 = 0 then: θ =........ °

(0 , 90 , 180 , 270)

20) If: $0° < \theta < 180°$, $\sqrt{3}$ Tan θ – 1 = 0 then: θ =..... °

(30 , 60 , 120 , 150)

21) The simplest form of: 1 + $Cot^2 \theta$ is...............

($Sin^2 \theta$, $Cos^2 \theta$, $Sec^2 \theta$, $Csc^2 \theta$)

22) The simplest form of: $Sin^2 \theta + Cos^2 \theta - Csc^2 \theta$ is......

(0 , 1 , $-Cot^2 \theta$, $Tan^2 \theta$)

23) The simplest form of: Sin(90 – θ) Csc(180 – θ)

is......... (–1 , 1 , Cot θ , Tan θ)

24) The general solution of the equation: Cos θ = 1 is......

($n\pi$, $2n\pi$, $\frac{\pi}{2} + n\pi$, $\frac{\pi}{2} + 2n\pi$)

25) The general solution of the equation: $\sin\theta - 1 = 0$ is...... $(\pi + 2\pi n \quad , \quad 2\pi n \quad , \quad \frac{\pi}{2} + n\pi \quad , \quad \frac{\pi}{2} + 2\pi n)$

26) If: $\sin\theta = \frac{1}{2}, \theta \in \left] \frac{\pi}{2}, \pi \right[$ then: $\theta =$.........

$(\frac{\pi}{6} + 2\pi \quad , \quad \frac{\pi}{3} + 2\pi \quad , \quad \frac{-\pi}{6} + \pi \quad , \quad \frac{-\pi}{6} + 2\pi)$

27) The general solution of the equation: $\tan\theta = \sqrt{3}$ is...... $(\frac{\pi}{3} + n\pi \quad , \quad \frac{\pi}{3} + 2n\pi \quad , \quad \frac{4\pi}{3} + 2n\pi \quad , \quad \frac{\pi}{6} + n\pi)$

28) $\frac{\tan\theta \cot\theta}{\sec\theta}$ =.... $(\sin\theta \quad , \quad \cos\theta \quad , \quad \sec\theta \quad , \quad \csc\theta)$

29) The perimeter of the circular sector =........

$(\frac{1}{2} r + \ell \quad , \quad \frac{1}{2} r\ell \quad , \quad 2r + \ell \quad , \quad r + \ell)$

30) $\sin^2 2\theta + \cos^2 2\theta$ =..... $(1 \quad , \quad -1 \quad , \quad 4 \quad , \quad 2)$

31) $\begin{vmatrix} x & 2 \\ 4 & x \end{vmatrix} = \begin{vmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{vmatrix}$, then: x =.........

$(3 \quad , \quad -3 \quad , \quad \pm 3 \quad , \quad 9)$

32) If: $AB = \begin{pmatrix} 4 & 5 \\ -1 & 3 \end{pmatrix}$, then: $B^t A^t$ =.........

$\left(\begin{pmatrix} -1 & 4 \\ 3 & 5 \end{pmatrix} , \begin{pmatrix} 4 & 5 \\ -1 & 3 \end{pmatrix} , \begin{pmatrix} 4 & -1 \\ 5 & 3 \end{pmatrix} , \begin{pmatrix} 4 & 3 \\ 5 & 1 \end{pmatrix}\right)$

33) If: $\begin{vmatrix} 2 - x & 2 \\ -3 & x + 2 \end{vmatrix} = 1$, then: x =.................

$(3 \quad , \quad -3 \quad , \quad \pm 3 \quad , \quad \pm 4)$

34) $(\text{Cos}\,\theta + \text{Sin}\,\theta)^2 - 2\text{Sin}\,\theta\,\text{Cos}\,\theta =$...... (1 , 2 , 3 , 0)

35) If: $\text{Sec}\,\theta - \text{Tan}\,\theta = \frac{2}{5}$, then: $\text{Sec}\,\theta + \text{Tan}\,\theta =$........

($\frac{2}{5}$, $\frac{5}{2}$, $-\frac{2}{5}$, $-\frac{5}{2}$)

36) The simplest form of: $\text{Sin}^2\,\theta + \text{Tan}^2\,\theta + \text{Cos}^2\,\theta$

is.......... ($\text{Sec}^2\,\theta$, 1 , $\text{Csc}^2\,\theta$, $\text{Tan}^2\,\theta$)

37) If: $0° < \theta < 180°$, $2\text{Cos}\,\theta + 1 = 0$ then: $\theta =$..... °

(300 , 240 , 210 , 120)

38) The S.S of: $-1 \le -x \le 1$ in R is..........

($]-1, 1[$, $R -]-1, 1[$, $\{0, 1\}$, $[-1, 1]$)

39) $\frac{1 - Sin^2\,\theta}{1 - Cos^2\,\theta} =$...... (1 , -1 , $\text{Tan}^2\theta$, $\text{Cot}^2\,\theta$)

40) The area of the equilateral triangle whose side length is x cm. equals............ cm²

(x^2 , $\frac{\sqrt{3}}{2}x^2$, $\frac{\sqrt{3}}{4}x^2$, $\frac{1}{2}x^2$)

41) The area of the square whose side length is x cm.

equals............ cm² (x^2 , $\sqrt{2}\,x^2$, $\frac{\sqrt{2}}{2}x^2$, $\frac{1}{2}x^2$)

42) The area of the regular octagon whose side length is x cm. equals........

($2x^2\,Cot\,45$, $2x^2Tan\,45$, $8\,x^2Cot\,22.5$, $2\,x^2Cot\,22.5$)

Third: answer the following questions:

1) If: $A = (A_{xy})\ \forall\ x , y \in \{1 , 2 , 3\}$ write the matrix A given that: $A_{xy} = y - x$ then find A^t

2) Find the area of Δ ABC where: $A(-4 , 2)$, $B(3 , 1)$, $C(-2 , 5)$ using determinants

3) Find the value of x if: $\begin{vmatrix} 1 & 0 & 0 \\ 1 & x & x \\ 5 & 2 & x \end{vmatrix} = 3$

4) Find the value of x if: $\begin{vmatrix} x & 0 & 0 \\ 4 & x & x \\ 3 & 2 & x \end{vmatrix} = 3x$

5) Is the matrix $A = \begin{pmatrix} 1 & -1 & 4 \\ -1 & 2 & 6 \\ 4 & 6 & 5 \end{pmatrix}$ Symmetric or skew symmetric?

6) Is the matrix $A = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 3 \\ 1 & -3 & 0 \end{pmatrix}$ Symmetric or skew symmetric?

7) If: $A = \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2d & -1 \\ 3e & 4 \end{pmatrix}$ where $A = B^t$ then find d and e

8) Find the S.S of the following equations using Cramer's method: $2x - 3y = 3$, $x + 2y = 5$

9) If: $A = \begin{pmatrix} 0 & 3x & 7 \\ x+3 & 0 & -2Z \\ 3y-x & 6 & 0 \end{pmatrix}$ is a skew symmetric matrix. Find the value of x , y , z

10) If: $A = \begin{pmatrix} 5 & 2x & 8 \\ -4 & -3 & 6 \\ x+2y & 6 & 4 \end{pmatrix}$ is a symmetric matrix. Find the value of x , y

11) Solve the S.S of the following equations using Cramer's method: $2x + y - 2Z = 10$, $3x + 2y + 2Z = 1$, $5x + 4y + 3Z = 4$

12) Using determinants to prove that the points A(3 , 5) , B(4 , − 1) , C(5 , − 7) are collinear

13) Find the area of Δ ABC where: A(−1 , −3) , B(2 , 4) , C(−3 , 5) using determinants

14) Find a , b and c if: $\begin{pmatrix} -2 & 3 \\ 4 & -1 \end{pmatrix} + \begin{pmatrix} a & b \\ c & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$

15) If: $A = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$ Find: $3A^{-1} + 5I$

16) If: $X^t + \begin{pmatrix} 2 & 1 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ find the matrix X

17) If: $A = \begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix}$ prove that: $A^2 - 2A - 3I = \square$

18) If: $A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$ then find X where

$2X^t - AB = \begin{pmatrix} 2 & -1 \\ -5 & 2 \end{pmatrix}$

19) If: $A = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$ then find $A^t B$

20) If: $A^t = \begin{pmatrix} 2 & -4 \\ 4 & 3 \end{pmatrix}$ prove that: $A^2 - 5A + 2I = \square$

21) Find the values of a , which make the matrix $\begin{pmatrix} a & 2 \\ 8 & a \end{pmatrix}$ has a multiplicative inverse.

22) Find the values of x , which make the matrix $\begin{pmatrix} x & 9 \\ 4 & x \end{pmatrix}$ has no multiplicative inverse.

23) If: $A = \begin{pmatrix} 2 & 0 \\ 3 & 4 \end{pmatrix}$, Prove that for the matrix A, there is a multiplicative inverse, then find it

24) Find a and b if: $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} a & 7 \\ 3 & b \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ 8 & 18 \end{pmatrix}^t$

25) If: $A = \begin{pmatrix} 1 & 4 \\ 3 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$ then: find AB

26) Solve the system of the following equations using the matrices: $3x + y = 2$, $5x + 4y = -6$

27) Solve the system of the following equations using the matrices: $3x + 2y = 5$, $2x + y = 3$

28) Use the matrices to find the two numbers in which their sum equal 10 and the difference between them equal 4.

29) Represent graphically the S.S of the following inequality $2x - 5y \geq 10$ in $\mathbb{R} \times \mathbb{R}$

30) Solve the following linear inequalities graphically: $3x + 5y \geq 15$, $y < x - 1$

31) Solve the following linear inequalities graphically: $x \geq 0$, $y \leq 2$, $2x + 3y \leq 12$

32) Use the linear programing, find each of the minimum value and the maximum value for the function $P = 4x + y$ under restrictions: $x \geq 0$, $y \geq 0$, $x + y \leq 6$, $2x + y \geq 10$

33) Use the linear programing, find each of the minimum value and the maximum value for the function $P = 3x + 2y$ under restrictions $x \geq 0$, $y \geq 0$, $x + y \leq 8$, $y \geq 3$

34) Find the greatest possible value of the function $P = 3x + 2y$ under the following restrictions:
$x \geq 0$, $y \geq 0$, $2x + y \leq 8$, $2x + 3y \leq 12$

35) Find the maximum value of the object function $P = 2x + y$ given that: $x \geq 0$, $y \geq 0$, $2x + 3y \leq 18$, $-4x + y \geq -8$

36) Find the minimum value of the object function $P = 3x + 2y$ given that: $x \geq 0$, $y \geq 0$, $2x + y \geq 8$, $x + 3y \geq 9$

37) Find the area of the circular sector whose perimeter equals 28 cm, and the length of the radius of its circle equals 8 cm.

38) A circular sector in which the measure of its angle equals 60° and the length of the radius of its circle equals 12 cm. Find its area to the nearest tenth.

39) Find the area of the regular octagon in which the length of its side equals 6 cm approximating the result to the nearest hundredth

40) Prove that: $\frac{\text{Cos}^2\theta}{1 - \text{Sin}\,\theta} = 1 + \text{Sin}\,\theta$

41) Prove that: $\frac{\text{Cot}\,\theta}{1 + \text{Cot}^2\theta} = \text{Sin}\,\theta\,\text{Cos}\,\theta$

42) Prove that: $\frac{1 + \text{Tan}^2\theta}{1 + \text{Cot}^2\theta} = \text{Tan}^2\theta$

43) Prove that: $\text{Tan}\,\theta + \text{Cot}\,\theta = \text{Sec}\,\theta\,\text{Csc}\,\theta$

44) Prove that: $\text{Sin}\,\theta\,\text{Sin}(90 - \theta)\,\text{Tan}\,\theta = 1 - \text{Cos}^2\,\theta$

45) Find the general solution: $\text{Cos}(\frac{\pi}{2} - \theta) = \frac{1}{2}$

46) Prove that: $\frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos^2\theta + \cos\theta\sin^2\theta} = \csc\theta - \sec\theta$

47) Prove that: $\frac{1}{1+\cot\theta} = \frac{\tan\theta}{1+\tan\theta}$

48) Prove that: $\frac{1+\tan^2\theta}{\sec^4\theta} = 1 - \sin^2\theta$

49) If: $\frac{3\cos\theta - 2\sin\theta}{3\cos\theta + 2\sin\theta} = \frac{1}{2}$ then find $\tan\theta$

50) Find the general solution: $\cos\theta = \frac{\sqrt{2}}{2}$

51) Find the general solution: $\tan\theta = \sqrt{3}$

52) Find the general solution: $\sqrt{2}\sin\theta\cos\theta - \sin\theta = 0$

53) Solve the equation: $\sin\theta\cos\theta = \frac{1}{2}\cos\theta$ where $0° < \theta < 180°$

54) Find the general solution: $\cos\theta = \sin 2\theta$

55) Find the general solution: $2\sin^2\theta = \sin\theta$

56) If $0° < \theta < 360°$ Find the solution set of equations:

$4\sin^2\theta - 3\sin\theta\cos\theta = 0$

57) Area of a circular sector is 270 cm^2 and the length of the radius of its circle equals 15 cm, find in radian the measure of its angle.

58) Find the S.S: $2\cos^2\theta - \cos\theta - 1 = 0$ where $\theta \in [0, 2\pi[$

59) Find the area of the circular sector in which the length of the radius of its circle is 10 cm and the measure of its angle is 1.2^{rad}

60) Find the area of the circular segment whose length of the radius of its circle equals 8 cm, and the measure of its central angle equals 150°.

61) Find the area of the circular segment whose length of the radius of its circle equals 10 cm and the measure of its circle equals 2.2^{rad} approximating the result the nearest hundredth.

62) Find the area of the circular segment in which the length of the radius of its circle is 8 cm and its height 4 cm

63) Circular segment in which its central angle 90° and its area = 56 cm^2. Find the length of its radius.

64) A chord of length 8 cm, in a circle is at a distance 3 cm from its center. Find the area of its circular segment

65) A circular sector whose perimeter equals 24 cm, and the length of its arc equals 10 cm. Find the area of its circle.

66) Circular segment in which the length of its chord = the length of its radius = 8 cm. Find its area

67) Solve the triangle ABC, right angled at B in which BC = 5 cm , AC = 13 cm

68) Solve the triangle ABC, right angled at B in each of the following cases:

A) AB = 8 cm , $m(\angle C) = 34^\circ$

B) AC = 26 cm , $m(\angle A) = 53^\circ 12'$

69) A circle of radius 7 cm, a chord was drawn in it opposite to a central angle of measure 110°. Calculate the length of this chord.

70) An equilateral triangle whose area = $9\sqrt{3}$ cm^2, find the length of its side.

71) Using determinants to prove that the points A(3 , 5) , B(4 , 6) , C(5 , 7) are collinear

72) A person observed the top of a hill 2.56 km from the point on the ground. He found its depression angle was 63°. Find the distance between the top and the observer to the nearest metre

73) ABC in which: AB = 8 cm , BC = 7 cm , AC = 11 cm, find its area

74) From the top of a tower 60 metres high, the angle of depression of a body located in a horizontal level which passes through the base of the tower equals $28^\circ\ 36'$. Find how far was the body from the base of the tower to the nearest metres.

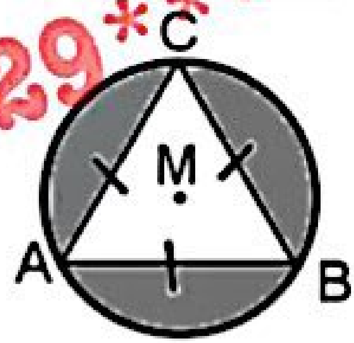
75) A person stands at 50 metres from the base of a tower. He observed the elevation angle of the top of the tower and found it to be $19^\circ 24'$. Find the height of the tower to the nearest metre.

76) A boat was observed from the top of the lighthouse of height 50 m. it was found that its depression angle 35°. Find the distance between the boat and the top of the lighthouse.

77) In the opposite figure:

ABC is an equilateral triangle drawn in circle M of radius equal 8 cm

find the area of each shaded circular segment



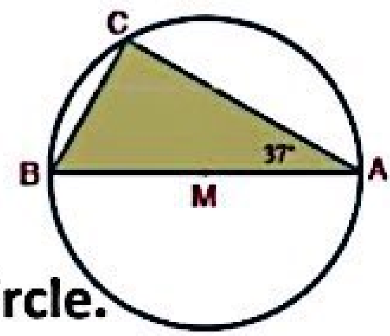
78) In the figure opposite:

circle M, $\overline{AB}$ is a diameter in it ,

AC = 12 cm, m(∠ A) = 37°

find the length of the radius of the circle.

to the nearest hundredth

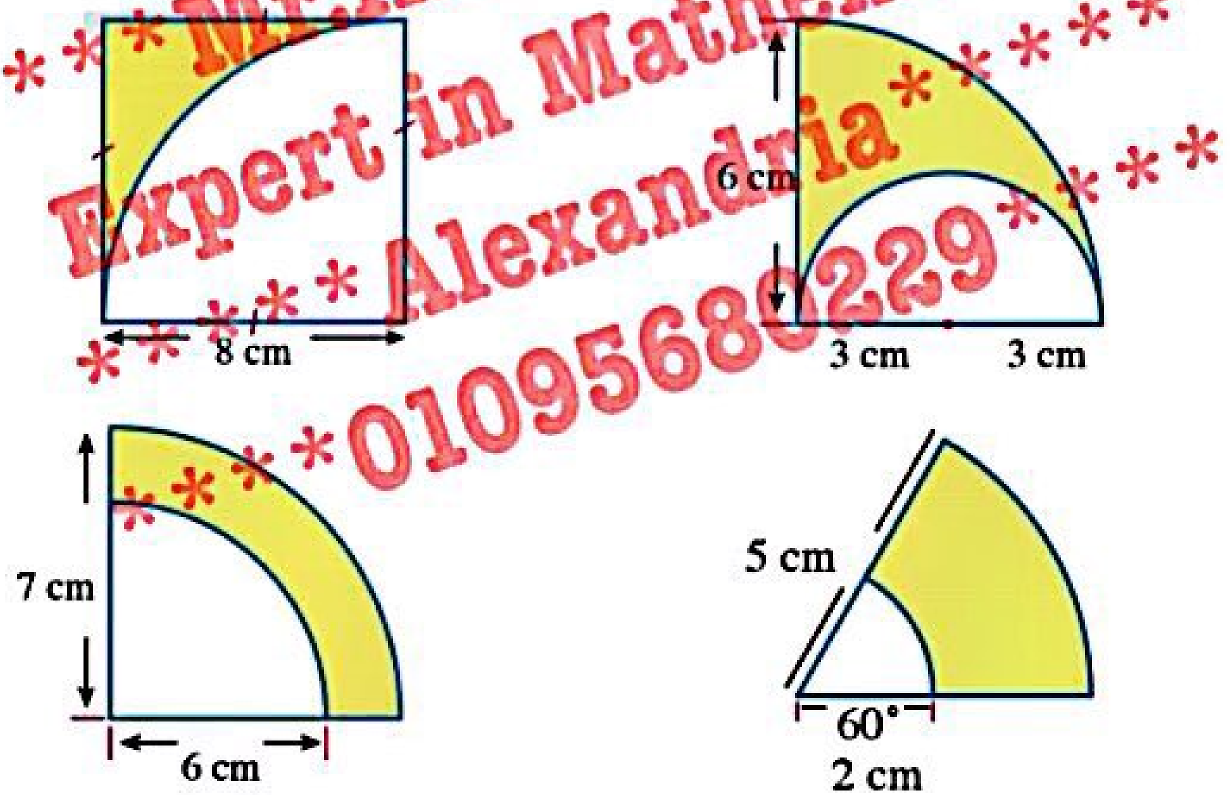


79) The curve whose equation: $y = a x^2 + b x$ passes through the two points (3 , 0) and (4 , 8) use the matrix to find the constants a and b

80) The straight line whose equation: $y + a x = c$ passes through the two points (1 , 5) and (2 , 1) use the matrix to find the constants a and b

81) ABC is an equilateral triangle of side length 24 cm drawn in a circle, find the radius length of the circle and the area of circular segment of the chord $\overline{BC}$

82) Find in terms of π the area of the shaded part in each of the following figures:



With my best wishes

Real life applications of linear programing

1) A small factory produces metal furniture 20 cupboard weekly at most of two different kinds A and B. If the profit from kind "A' is 80 pounds, and profit from kind B is 100 pounds. The factory sells from kind A at least 3 times what it sells from the second kind. Find number of cupboard from each kind to satisfy greatest possible profit to the factory.

2) Consumer: Two package of food substances, the first gives 3 calories and has 5 units of vitamin "C", the second gives 6 calories and has 2 units of vitamin "C', given that we need at least 36 calories and at least 25 units of vitamin "C". The price of the unit of the first article is 6 pounds, and of the second is 8 pounds. Find the number of each article that should be bought to obtain what we need at the least cost.

" اللهم صل وسلم وبارك على سيدنا محمد وعلى آله وصحبه وسلم "

[A] Choose the correct Answer :

First Geometry Rules

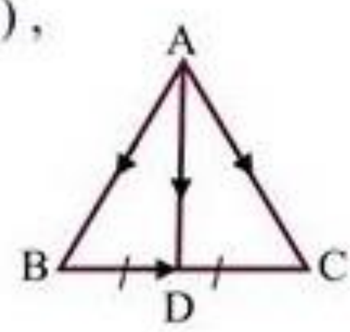
(1) The norm of the vector $\overrightarrow{A}$ (x, y): $\| A \| = \sqrt{x^2 + y^2}$

(2) The polar form from the position vector: $(\| A \|, \theta^\circ)$, $\tan \theta = \frac{y}{x}$, $x = \| A \| \cos \theta$, $y = \|A\| \sin \theta$

(3) The vector $\overrightarrow{A}$ in fundamental form $A = x \overrightarrow{i} + y \overrightarrow{j}$

(4) If : $\overrightarrow{A} = (x_1, y_1)$, $\overrightarrow{B} = (x_2, y_2)$, then :

- $\overrightarrow{A} + \overrightarrow{B} = (x_1 + x_2 +, y_1 + y_2)$
- $\overrightarrow{A} // \overrightarrow{B} \supset x_1 \times y_2 - x_2 \times y_1 = 0$, **or** $m_1 = m_2$ (m slope)
- $\overrightarrow{A} \perp \overrightarrow{B} \supset x_1 \times x_2 + y_1 \times y_2 = 0$, or $m_1 \times m_2 = -1$
- $\overrightarrow{AB} = B - A = (x_2 - x_1, y_2 - y_1)$,

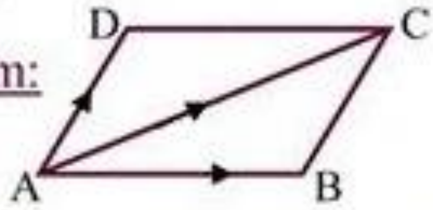


(5) In any triangle ABC :

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$, $\overrightarrow{AB} = -\overrightarrow{BA}$

$\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$

In a parallelogram:



$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$

In Quadrilateral: $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = 0$

(6) Physical application :

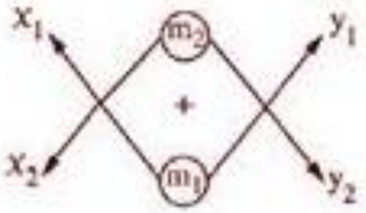
- The resultant force $(\overrightarrow{F}) = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \dots\dots\dots$

  If $\overrightarrow{F} = 0$, the system is equilibrium

- The relative velocity $\overrightarrow{V_{AB}} = \overrightarrow{V_B} - \overrightarrow{V_A}$

(7) Division of a line segment:

$A = (x_1, y_1)$, $B = (x_2, y_2)$, $C(x, y)$ divide $\overline{AB}$ by ratio $\frac{m_2}{m_1}$



- Internally:

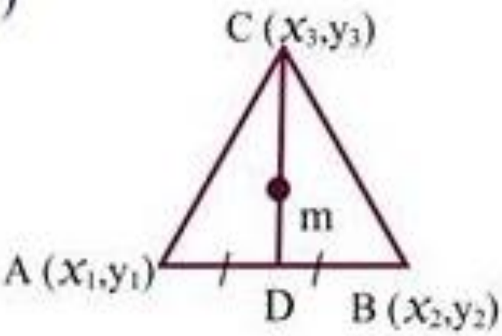
$C(x, y) = \left(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}, \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} \right)$

- Externally:

$C(x, y) = \left(\frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}, \frac{m_1 y_1 - m_2 y_2}{m_1 - m_2} \right)$

- Midpoint:

$D(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$



- Point of concurrence:

$m(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$

(8) To prove that the three points A , B , C are collinear using vectors, Prove that $\overrightarrow{AB} = K\,\overrightarrow{AC}$

(9) To prove the quadrilateral ABCD is a parallelogram we prove that : $\overrightarrow{AB} = \overrightarrow{DC}$ or $\overrightarrow{AD} = \overrightarrow{BC}$

(10) To prove that the parallelogram is **rectangle**:

We prove that: $\overrightarrow{AB} \perp \overrightarrow{BC}$ or $\|\overrightarrow{AC}\| = \|\overrightarrow{BD}\|$

(11) To prove that the parallelogram is **rhombus**:

We prove that: $\|\overrightarrow{AB}\| = \|\overrightarrow{BC}\|$ or $\overrightarrow{AC} \perp \overrightarrow{BD}$

(12) <u>The equations of straight line:</u>

If : $\overrightarrow{A} = (x_1 , y_1)$, Vector $\overrightarrow{u} = (a , b)$ then :

- The vector equation: $\vec{r} = \overrightarrow{A} + k\,\overrightarrow{u} = (x_1 , y_1) + k\,(a , b)$

  $$m \text{ (slope)} = \frac{b}{a}$$

- The two parametric equations are :

  $$X = x_1 + k\,a \qquad , \qquad Y = y_1 + k\,b$$

- Cartesian equation: $\frac{Y - y_1}{X - x_1} = m$, $m = \frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$

- In terms of the slope (m) and the intercept c

  $$y = m\,x + c$$

- In terms of the two intercepts a , b from the x–axis and the y–axis respectively: $\frac{x}{a} + \frac{y}{b} = 1$

- General equation of the straight line:

  $$a_1\,x + b_1\,y + c_1 + k\,(a_2\,x + b_2\,y + c_2) = 0$$

(13) Measure of angel between two straight lines :

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|, \theta \in [\,0\,, \frac{\pi}{2}\,[\,,\ m_1 = \tan\theta_1\,,\ m_2 = \tan\theta_2$$

(14) The length of the perpendicular from a point to a straight line

Point $(x_1\,, y_1)$, equation of St. line $a x + b\,y + c = 0$

$$L = \frac{|a\,x_1 + b\,y_1 + c|}{\sqrt{a^2 + b^2}}$$

Important Notice

- The slope of straight line parallel to X –axis = Zero
- The slope of straight line parallel to Y –axis is undefined
- $L_1 \,//\, L_2 \quad \Rightarrow \quad m_1 = m_2$
- $L_1 \perp L_2 \quad \Rightarrow \quad m_1 \times m_2 = -1$
- If slope of $\overrightarrow{AB}$ = slope of $\overrightarrow{AC}$, then A , B , C are collinear
- If the straight line parallel to X-axis :
  - Vector $\overrightarrow{u} = (1, 0)$, Cartesian equation is $Y = y_1$
- If the straight line parallel to Y-axis :
  - Vector $\overrightarrow{u} = (0, 1)$, Cartesian equation is $X = x_1$
- The Cartesian equation of straight line passing through origin point (0,0) is Y = m X , m (slope)

Second Exercises

[A] Choose the correct answer from those given: -

(1) If $\overrightarrow{A} = 2\hat{i} + 3\hat{j}$, $\overrightarrow{B} = (4 , k)$ and $\overrightarrow{A} \parallel \overrightarrow{B}$, then k = ...

(a) 6 (b) – 6 (c) 3 (d) 12

(2) The length of perpendicular from (0 , 6) to the line $X = 2$ is .... unit of length.

(a) 1 (b) 2 (c) 6 (d) 4

(3) If $\overrightarrow{A} = 2\hat{i} + 3\hat{j}$, $\overrightarrow{B} = 5\hat{i} + 7\hat{j}$, then $\|\overrightarrow{AB}\|$ = ...

(a) 1 (b) 25 (c) 7 (d) 5

(4) The measure of the angle between the two straight lines : $3X - 7 = 0$, $y = 5$ is ....°

(a) 0 (b) 180 (c) 90 (d) 45

(5) If : A = (3 , 4) , B = (– 1 , 3) , then $\|\overrightarrow{AB}\|$ = ...

(a) 5 (b) 17 (c) $\sqrt{5}$ (d) $\sqrt{17}$

(6) The length of the perpendicular drawn from the point (1 , – 1) to the straight line whose equation is : $X - y = 0$, is .... length units.

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

(7) Which of the following straight lines makes an angle of measure $\frac{3\pi}{4}$ with the positive direction of X-axis ....

(a) $X + y = 6$ (b) $y - X = 6$

(c) $y + \sqrt{2}\,X = 6$ (d) $y - \sqrt{2}\,X = 6$

(8) The measure of the acute angle included between the two straight lines : $y = -X$, $X = 0$ ....

(a) 30° (b) 60° (c) 45° (d) 90°

(9) If : C (2 , 0) is a midpoint of $\overline{AB}$, where A (3 , 7) , then B = ...

(a) (– 1 , 7) (b) (3 , 0) (c) (1 , – 7) (d) (2.5 , 3.5)

(10) The direction vector of the straight line : $3X - 7y + 5 = 0$ is ....

(a) (3 , 7) (b) (– 3 , 5) (c) (7 , 3) (d) (5 , 7)

(11) If : $\overrightarrow{AB} = \overrightarrow{CD}$, $\overrightarrow{AB} = (6 , 4)$, $\overrightarrow{C} = (-1 , 3)$, then $\overrightarrow{D}$ = ...

(a) (5 , 7) (b) (– 5 , – 7) (c) (– 5 , 7) (d) (7 , 1)

(12) If : A (3 , 5) , B (– 1 , k) , $\|\overline{AB}\| = 4$, then k = ...

(a) 0 (b) 5 (c) 10 (d) ± 5

(13) The length of the perpendicular drawn from the point (– 3 , 5) to y-axis equals ....

(a) 2 (b) 3 (c) 5 (d) 8

(14) The equation of the straight line which passes through the point $(2 , -3)$ and parallel to x-axis is ....

(a) $x + 3 = 0$ (b) $y + 3 = 0$ (c) $x - 2 = 0$ (d) $y - 3 = 0$

(15) The vector : $6\,\overline{i} - 6\,\overline{j}$ is expressed in the polar form by the vector : ....

(a) $\overrightarrow{m} = \left(6 , \frac{3\pi}{4}\right)$ (b) $\overrightarrow{m} = \left(6\sqrt{2} , \frac{3\pi}{4}\right)$

(c) $\overrightarrow{m} = \left(6\sqrt{2} , \frac{5\pi}{4}\right)$ (d) $\overrightarrow{m} = \left(6\sqrt{2} , \frac{7\pi}{4}\right)$

(16) If : $\overrightarrow{A} = (-1 , 5)$, $\overrightarrow{B}(2 , 1)$, then $\|\overrightarrow{AB}\| =$ .... length units.

(a) 9 (b) 16 (c) 5 (d) 25

(17) If : $\overrightarrow{C} = \left(8 , \frac{2\pi}{3}\right)$ is a position vector of the point C with respect to the origin point O , then the coordinates of C is ....

(a) $\left(4 , 4\sqrt{3}\right)$ (b) $\left(-4 , 4\sqrt{3}\right)$ (c) $\left(4\sqrt{3} , -4\right)$ (d) $\left(-4 , -4\sqrt{3}\right)$

(18) The length of the perpendicular from the point $(1 , 1)$ to the straight line $x + y = 0$ equals .... length unit.

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

(19) If : $\overrightarrow{AB} = \overrightarrow{CD}$, where $\overrightarrow{AB} = (6 , 4)$, $\overrightarrow{C} = (-1 , 3)$, then $\overrightarrow{D} =$ ....

(a) $(5 , 7)$ (b) $(-5 , 7)$ (c) $(-5 , -7)$ (d) $(7 , 7)$

(20) If the straight line : $3x + 4y - 24 = 0$ intersect with the two coordinates axes x and y in the two points A , B respectively where O is the origin point , then the area of Δ OAB = ... square unit.

(a) 48 (b) 24 (c) 12 (d) 6

(21) If : $A = (1 , 3)$, $B = (2 , 5)$, $C = (-3 , -7)$, $\overrightarrow{AB} = \overrightarrow{CD}$, then D is ....

(a) $(2 , 5)$ (b) $(2 , -5)$ (c) $(-2 , -5)$ (d) $(-2 , 5)$

(22) If : $\overrightarrow{A} = (2 , -3)$ is a direction vector to a straight line , then .... is a direction vector to the same straight line.

(a) $(-2 , 3)$ (b) $(-2 , -3)$ (c) $(2 , 3)$ (d) $(-6 , -9)$

(23) The Cartesian equation of the straight line which passes through the point $(3 , -4)$ and the direction vector to it is $(2 , -1)$ is ....

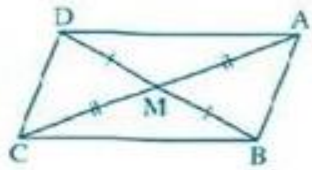
(a) $x + 2y + 5 = 0$ (b) $2x + y - 5 = 0$

(c) $x - 2y - 5 = 0$ (d) $x - 2y + 5 = 0$

(24) All statements express $\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} + \overrightarrow{MD}$ except : ....

(a) $\overrightarrow{AB} + \overrightarrow{DC}$ (b) $\overrightarrow{AB} + \overrightarrow{BM} + \overrightarrow{MA}$

(c) $\overrightarrow{O}$ (d) $\overrightarrow{AB} + \overrightarrow{CD}$



(25) If : A = (3 , 8) , B = (– 3 , 0) , then : $\|\overrightarrow{AB}\|$ ....

(a) 8 (b) 10 (c) ± 8 (d) ± 10

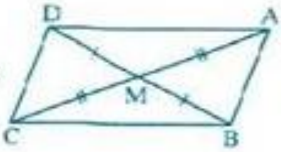
(26) The length of the perpendicular drawn from the point (0 , – 5) to the straight line : $x + 7 = 0$ equals ....

(a) 2 (b) 5 (c) 7 (d) 12

(27) If $\vec{u} = (2 , 3)$ is a direction vector to a line, then the perpendicular to it is ....

(a) (3 , – 2) (b) (3 , 2) (c) (– 2 , 3) (d) (5 , 3)

(28) **In the opposite figure :**



All the following statments expresses $\overrightarrow{AC}$ except the statement :

(a) $2\,\overrightarrow{AM}$ (b) $\overrightarrow{AD} + \overrightarrow{DC}$

(c) $\overrightarrow{AB} + \overrightarrow{BD}$ (d) $\overrightarrow{BC} + \overrightarrow{DC}$

(29) In Δ ABC : $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} =$ ....

(a) $\overrightarrow{AB}$ (b) $\overrightarrow{BC}$ (c) $\overrightarrow{CA}$ (d) $\vec{0}$

(30) If : $\vec{u} = (3 , 2)$ is the direction vector of a straight line , then the perpendicular direction vector of the straight line is ....

(a) (– 2 , 3) (b) (6 , 4) (c) (– 6 , 4) (d) $\left(\frac{1}{3} , \frac{1}{2}\right)$

(31) The straight line : $x + 3y = 0$ ....

(a) parallel to x-axis. (b) parallel to y-axis.

(c) passes through the origin point. (d) parallel to straight line $3x + y = 0$

(32) If : $\vec{A} = (2 , 5)$ and $\vec{B} = (K , -4)$ and $\vec{A} \perp \vec{B}$, then K = ...

(a) 2 (b) 5 (c) – 4 (d) 10

(33) If $\overrightarrow{OC} = \left(8 , \frac{2\pi}{3}\right)$ is the position vector of the point C relative to the origin point O , then the coordinates of the point C are ....

(a) $\left(4 , 4\sqrt{3}\right)$ (b) $\left(-4 , 4\sqrt{3}\right)$ (c) $\left(4\sqrt{3} , -4\right)$ (d) $\left(-4 , -4\sqrt{3}\right)$

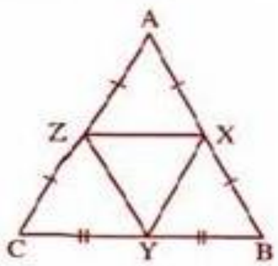
(34) If : $\vec{u} = (2 , -3)$ is the direction vector of a straight line, then all of the following vectors are direction vectors to the same straight line except the vector ....

(a) (– 2 , 3) (b) (– 2 , – 3) (c) (4 , – 6) (d) (– 4 , 6)

(35) **In the opposite figure :**

all the following statement :

Express $\|\overrightarrow{XZ}\|$ except = ...



(a) $\|\overrightarrow{XY} + \overrightarrow{YZ}\|$ (b) $\|\overrightarrow{ZY} + \overrightarrow{YX}\|$

(c) $\frac{1}{2}\,\|\overrightarrow{BC}\|$ (d) $\|\overrightarrow{XB} + \overrightarrow{BY}\|$

(36) Which of the following straight lines passes through the origin point ....

(a) $2x+3=0$ (b) $x+3y=0$ (c) $2x+3y=12$ (d) $y-5=0$

(37) The polar form of the position vector of the point $A\left(6, 6\sqrt{3}\right)$ with respect to the origin point is ....

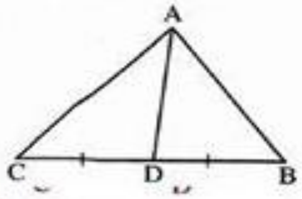
(a) $(12, 60°)$ (b) $(12, 30°)$ (c) $(10, 60°)$ (d) $(10, 30°)$

(38) **In the opposite figure :**

$2\overrightarrow{AD}=$ ....

(a) $2\overrightarrow{AB}+2\overrightarrow{CD}$ (b) $\overrightarrow{AB}+\overrightarrow{BD}$

(c) $\overrightarrow{AB}+\overrightarrow{AC}$ (d) $\overrightarrow{BA}+\overrightarrow{CA}$



(39) If $C(2, 4)$ is the midpoint of $\overrightarrow{AB}$ where $A(x, 4)$, $B(1, y)$ ....

(a) $x=3, y=4$ (b) $x=4, y=3$

(c) $x=2, y=6$ (d) $x=0, y=0$

(40) If $\hat{u}=\left(\frac{1}{2}, 1\right)$ is a direction vector to the line, then all the following vectors are perpendicular to the line except the vector : ....

(a) $\left(1, -\frac{1}{2}\right)$ (b) $(2, -1)$ (c) $\left(-1, -\frac{1}{2}\right)$ (d) $(4, -2)$

(41) If : $\overrightarrow{A}=3\hat{i}-4\hat{j}$, then $\|2\overrightarrow{A}\|=$ ....

(a) 5 (b) 3 (c) -4 (d) 10

(42) The length of the perpendicular from the point $(3, -4)$ to the x-axis = ...

(a) 3 (b) -4 (c) 5 (d) 4

(43) If : $A=(2, 3)$, $B=(5, 4)$, then $\overline{AB}=$ ....

(a) $(1, 3)$ (b) $(-3, 1)$ (c) $(-1, 3)$ (d) $(3, 1)$

(44) If θ is the angle between L_1, and L_2, and $\tan\theta=-1$, then $m(\theta)=$ ....

(a) 135° (b) 145° (c) 90° (d) zero

(45) The measure of the angle between the two straight lines whose equations are $x=5$, $y+3=0$ equals : ....

(a) 30° (b) 45° (c) 60° (d) 90°

(46) Length of the perpendicular from the point $(1, 1)$ to the straight line whose equation $x+y=0$ equals ....

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

(47) If : $C=(2, 4)$ is the midpoint of $\overline{AB}$ where $A(3, y)$, $B(1, y)$, then $y=$ ....

(a) 1 (b) 2 (c) 3 (d) 4

(48) The length of the intercepted part from the x-axis by the straight line whose equation : $2x+3y=6$ is .... length unit.

(a) 2 (b) 3 (c) 4 (d) 6

(49) Let $A=(2, -2)$ and $B=(5, 2)$, then $\|\overrightarrow{AB}\|=$ .... length unit.

(a) 5 (b) 3 (c) 25 (d) 7

(50) Let $\overrightarrow{A} = (-2 , 4)$ and $\overrightarrow{B} = (6 , 3k)$, $\overrightarrow{A} \parallel \overrightarrow{B}$, then k = ...

(a) 4 (b) – 4 (c) 2 (d) – 2

(51) The equation of the straight line which passes through the point (2 , – 3) and parallel to the x-axis is ....

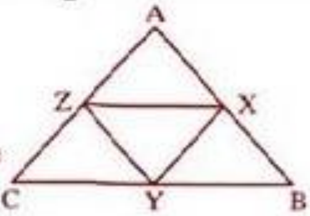
(a) $x + 3 = 0$ (b) $y + 3 = 0$ (c) $x - 2 = 0$ (d) $y - 3 = 0$

(52) The straight lines whose vector equation is $\overrightarrow{r} = (2 , -1) + k (3 , -5)$, its slope = ...

(a) $\frac{1}{2}$ (b) $\frac{-5}{3}$ (c) $\frac{-3}{5}$ (d) $\frac{-1}{2}$

(53) **ose the correct answer from the given ones :**

In the opposite figure :



AB = AC , X , Y , Z are the midpoints of sides of the triangle ABC Which of the following statements is true ?

(a) $\| \overrightarrow{XY} \| = \| \overrightarrow{ZY} \|$ (b) $\overrightarrow{XY}$ equivalent $\overrightarrow{ZY}$

(c) $\overrightarrow{BY}$ equivalent $\overrightarrow{ZX}$

(54) The vector $-12\hat{i} - 12\hat{j}$ is represented by the vector .... in the polar form.

(a) $\overrightarrow{m} = \left(12 , \frac{\pi}{4}\right)$ (b) $\overrightarrow{m} = \left(12\sqrt{2} , \frac{\pi}{4}\right)$

(c) $\overrightarrow{m} = \left(12\sqrt{2} , \frac{3\pi}{4}\right)$ (d) $\overrightarrow{m} = \left(12\sqrt{2} , \frac{5\pi}{4}\right)$

(55) If : $\hat{j} = (2 , -3)$ is the direction vector of a straight line, then all of the following are direction vectors for the same straight line except ....

(a) (– 2 , 3) (b) (– 2 , – 3) (c) (4 , – 6) (d) (– 4 , 6)

(56) Which of the following straight lines passes through the origin point ?

(a) $2x + 3 = 0$ (b) $x + 3y = 0$ (c) $2x + 3y = 12$ (d) $y - 5 = 0$

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End of the questions – Good luck

Dakahlia Governorate | Geometry | 1^{st} sec

Math's Supervision | 2^{nd} term 2017 | Time : 2 hour

Answer the following questions

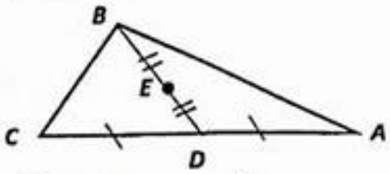
1) Complete :

a) If $A = (6, 0)$, $B = (0, 8)$, then $\|\overrightarrow{AB}\|$ = ------------------------

b) The length of the perpendicular drawn from the point $(2, 2)$ to St. line whose equation $x - y = 0$ is ----------------------------------

c) Using the figure

$\overrightarrow{BA} + \overrightarrow{BC} = ------ \overrightarrow{EB}$



d) The measure of the angle between the two St. lines $L_1 : 2x + y = 5$, and $L_2 : \vec{r} = (0,1) + t(2,-1)$ equals -------------------

2) Choose the correct answer :

a) The vector equation of St. line $4x + 3y = 12$ is ------------------

1) $\vec{r} = (-4,6) + t(4,3)$ 2) $\vec{r} = (6,-4) + t(3,4)$

3) $\vec{r} = (6,-4) + t(-3,4)$ 4) $\vec{r} = (6,-4) + t(-3,-4)$

b) if the vector $\vec{A} = (4, 240^o)$, $\vec{B} = (4, 60^o)$, then $\vec{A} - \vec{B}$ = ---

1) $(8, 180^o)$ 2) $(8, 60^o)$ 3) $(0, 180^o)$ 4) $(8, 240^o)$

c) If $\vec{A} = (1,2)$, $\vec{B} = (k, -4)$ and $\vec{A} \perp \vec{B}$, then k =--------

1) 2 2) 8 3) - 2 4) - 8

d) If $A = (7, 5)$, $C = (5, -2)$, then $\overrightarrow{AB} + \overrightarrow{BC}$ = ---------------

1) $(-2, -7)$ 2) $(2, 3)$ 3) $(2, 7)$ 4) $(7, -2)$

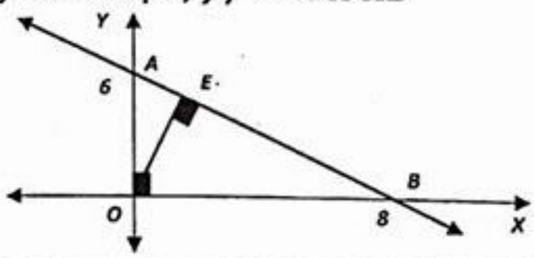
3) A) If $A = (3, -2)$, $B = (-2, 4)$ find the ratio by which $C(8, y)$ divides $\overline{AB}$, then deduce the value of y

B) In the opposite fig :

1) Find the vector equation of $\overleftrightarrow{AB}$

2) If $\overline{OE} \perp \overline{AB}$ find the equation of $\overleftrightarrow{OE}$

3) Find the length of $\overline{OE}$



4) A) Prove that the two St. line $\vec{r} = (0,4) + t(1,-2)$, $2x + y + 5 = 0$ are parallel then find the shorter distance between them

B) If the measure acute angle between two St. line $3x - 5y - 1 = 0$ and $kx - y = 3$ is $\frac{\pi}{4}$ Find the value of k

5) A) Find the equation of St. line which passes through the point $(3, 1)$ and the intersection point of the two lines $3x + 2y - 7 = 0$, $x + 3y = 7$

B) ABCD is a parallelogram , Y the intersection point of the two diagonals , and X is a point on its plane . prove that : $\overrightarrow{XA} + \overrightarrow{XB} + \overrightarrow{XC} + \overrightarrow{XD} = 4\,\overrightarrow{XY}$

Areas

Circular Sector

$$\text{Area} = \frac{1}{2}\,\theta^{\text{rad}}\,r^2$$

$$\text{Area} = \frac{X}{360} \times \pi\, r^2 = \frac{X}{360} \times \text{Area of the Circle}$$

$$\text{Area} = \frac{1}{2} \times L \times r$$

$$\text{Perimeter} = 2\,r + L$$

Circular Segment

$$\text{Area} = \frac{1}{2}\, r^2 (\,\theta^{\text{rad}} - \text{Sin}\,\theta\,)$$

$$\text{Perimeter} = \text{Arc} + \text{Chord}$$

Triangle

$$\text{Area} = \frac{1}{2}\,\text{Base} \times \text{Height}$$

$$\text{Area} = \frac{1}{2}\,L1 \times L2 \times \text{Sin}\,(\text{ included Angle })$$

$$\text{Area} = \sqrt{S(S-L1)(S-L2)(S-L3)} \quad \text{Where Perimeter} = 2\,S$$

Convex Quadrilateral

$$\text{Area} = \frac{1}{2}\,D1 \times D2 \times \text{Sin}\,(\text{ included Angle })$$

Rhombus

$$\text{Area} = L \times H$$

$$\text{Area} = \frac{1}{2}\,D1 \times D2$$

Square

$$\text{Area} = \frac{1}{2}\,D^2$$

$$\text{Area} = L^2$$

Regular Polygon :

$$\text{Area} = \frac{1}{4}\,nX^2\,\text{Cot}\,\frac{\pi}{n} \quad \text{where } n : \text{number of sides}\ ,\ X : \text{Side Length}$$

Equilateral triangle :

$$\text{Area} = \frac{\sqrt{3}}{4} X^2 \qquad \text{Where : X : Side Length}$$

Regular Hexagon :

$$\text{Area} = \frac{3\sqrt{3}}{2} X^2 \qquad \text{Where : X : Side Length}$$

LESSON 2 Vectors

The norm of the vector : $\| A \| = \sqrt{X^2 + Y^2}$, Where : A (X , Y)

The polar form from the position vector : (Norm $\| A \|$, Angle :θ)

If : $A = (X_1 , Y_1) , B = (X_2 , Y_2)$, then :

$A + B = (X_1 + X_2 + , Y_1 + Y_2)$

$A \parallel B$ if : $X_1 \times Y_2 - X_2 \times Y_1 = 0$, or $m_1 = m_2$ ($slope_1 = slope_2$)

$A \perp B$ if : $X_1 \times X_2 + Y_1 \times Y_2 = 0$, or $m_1 \times m_2 = m_2 = (- 1) , (- 1) \div m_2$

$AB = B - A = (X_2 - X_1 , Y_2 - Y_1)$, Where : $A (X_1 , Y_1) , B (X_2 , Y_2)$

LESSON 2 Equation of the straight line

If : Point $A = (X_1 , Y_1)$, Vector $u = (a , b)$ then :

The vector equation is : $r = A + k u = (X_1 , Y_1) + k (a , b)$

The two parametric equations are : $X = X_1 + k a$, $Y = Y_1 + k b$

Cartesian equation is : $\frac{Y - Y_1}{X - X_1} = m (\frac{Y_2 - Y_1}{X_2 - X_1})$, where m (slope = $\frac{b}{a}$)

The slope = $m = \frac{Y_2 - Y_1}{X_2 - X_1} = \tan \theta$,

Find the Cartesian equation of the straight line passing through the point (3 , – 4) and makes an angle of measure 45° with the positive direction of the X-axis.

Two fundamental unit vectors = (magnitude Cos θ i , magnitude Sin θ j)

AB + BC = AC AB – AC = CB

AB = – BA , AC = – CA , AD = – DA , BC = – CB , CD = – DC ,

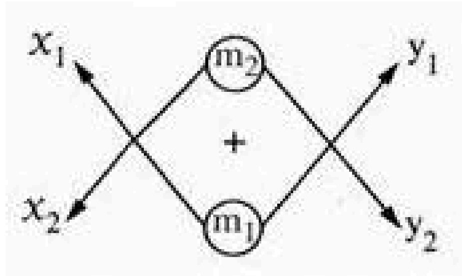
In any triangle : AB + BC + CA = 0 , In Quad : AB + BC + CD + DA = 0

$C (X , Y) , A = (X_1 , Y_1) , B = (X_2 , Y_2)$, then : ratio : $m_2 : m_1$

LESSON 1 Division of a line segment

Internally :

$$C(X, Y) = \left(\frac{m_1X_1 + m_2X_2}{m_1 + m_2}, \frac{m_1Y_1 + m_2Y_2}{m_1 + m_2}\right)$$



Externally :

$$C(X, Y) = \left(\frac{m_1X_1 - m_2X_2}{m_1 - m_2}, \frac{m_1Y_1 - m_2Y_2}{m_1 - m_2}\right)$$

Midpoint :

$$C(X, Y) = \left(\frac{X_1 + X_2}{2}, \frac{Y_1 + Y_2}{2}\right)$$

LESSON 3 Measure of the angle between two straight lines

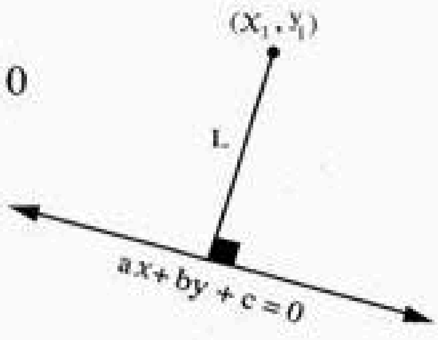
$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \quad \text{where } \theta \in \left[0, \frac{\pi}{2}\right[$$

$$, m_1 = \tan\theta_1 \text{ and } m_2 = \tan\theta_2$$

LESSON 4 The length of the perpendicular from a point to a straight line

- The length of the perpendicular (L) drawn from the point (x_1, y_1) to the straight line whose equation is : $ax + by + c = 0$ is determined by the relation :

The length of the perpendicular (L) $= \dfrac{|a x_1 + b y_1 + c|}{\sqrt{a^2 + b^2}}$



LESSON 5 General equation of the straight line passing through the point of intersection of two lines

$a_1 x + b_1y + c_1 + k(a_2 x + b_2y + c_2) = 0$, where k is a non-zero constant.